CAN THE LABOR MATCHING MODEL EXPLAIN LABOR MARKET MOVEMENTS?

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Abstract. The Mortensen-Pissarides model possesses two elements absent from the commonly used simplified version: the job destruction margin and training costs. The interaction between these elements enables a model driven by a single aggregate shock to fully account for labor market dynamics in the U.S. The role of endogenous job destruction is to create an additional pool of unemployed at the onset of a recession. The role of training costs is to explain the decline in vacancies and slow response of job creation. An extension of the model allows for search on the job and explains the behavior of quits.

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I. Introduction

The labor search model pioneered by Mortensen and Pissarides (1994, 1998), MP henceforth, has become the main framework for studying business cycle fluctuations in the labor market because of its intuitive explanation of equilibrium unemployment. Shimer (2005) argued that a calibrated version of the MP model cannot account for business cycle fluctuations of the magnitude observed in the U.S. This became known as the Shimer puzzle and spurred the development of a large number of increasingly elaborated models of labor market fluctuations.

I show that when the labor matching model allows for endogenous variations in the job destruction margin, and when a rich structure of costs is considered, the model can fully account for labor market fluctuations in the U.S. My main finding is that a reasonably parameterized labor matching model can simultaneously explain most of fluctuations in unemployment, vacancies, job destruction and job finding rates, all as a result of a single aggregate shock. The model is consistent with empirical volatility and cyclicality of productivity, real wages and profits, and generates mild responses of job creation.

Both the job destruction margin and training costs are key to the empirical performance of the model. Explaining variations in job destruction enhances the ability of the model to capture the behavior of unemployment, accounting for its initial increase during recessions. Incorporating training costs is crucial for explaining the decrease in vacancies and the modest response of job creation.

The contribution of this paper is to emphasize the interaction of these two key elements. The introduction of training costs in a model with job heterogeneity and endogenous job destruction does not substantially reduce the size of the surplus. Nor does it significantly amplify fluctuations in the size of match surplus. Instead, the role of training costs is to reduce the incentives of firms to post vacancies in response to endogenous increases in job destruction in a recession. This shows that the joint effect of the two elements is greater than the sum of their effects when taken separately. The interaction between the two key elements of the model constitutes a mechanism that provides a solution to the Shimer puzzle. It alters the responses of most labor market variables to aggregate shocks making them consistent with observed dynamics of these variables.

To clarify the dependence of labor market dynamics on various parameters, I derive a closed-form expression for the slope of the Beveridge curve. It shows how the elasticity of the matching function, the ratio of training to recruiting costs and other parameters jointly determine the slope of the Beveridge curve. The closed-form expression for the Beveridge curve helps illustrate the main forces which Shimer's model abstracts from. It demonstrates why both elements of the model, which differentiate it from Shimer's specification, and their interaction are crucial for model fit.

First, the ability of a firm and a worker to terminate their relationship based on match profits makes firms more eager to destroy jobs when aggregate conditions are worse and the value of a match is lower. The role of the job destruction margin in propagating aggregate shocks is to immediately create an additional pool of unemployed at the onset of a recession. As Shimer predicted, this element alone leads to a counter-factual positive correlation between unemployment and vacancies and a positively sloped Beveridge curve.

Second, job creation costs in my model are a mix of recruiting and training costs. Recruiting costs are the costs associated with opening and filling a vacancy, while training costs include all costs specific to the new match incurred conditional on finding a worker to fill the vacancy.[1] When more jobs are destroyed and the labor market becomes less tight, it is much easier for firms to find workers. In the absence of training costs, this would lead to an increase in the number of vacancies to preserve market tightness. The introduction of training costs attenuates the sensitivity of the total cost of creating a new job to variations in market tightness. Firms facing a lower value of a prospective match and a relatively small decrease in the cost of hiring choose to post fewer vacancies and create fewer new jobs, thus, reinstating the negatively sloped Beveridge curve. The dual structure of job creation costs explains

[1]The idea that creation costs can be a mix of vacancy-specific and match-specific costs was recently revived and discussed by Pissarides (2009). Non-linear creation costs were also used by Yashiv (2006); Rotemberg (2006).

the decrease in vacancies and the mild response of job creation once jobs have been destroyed without amplifying fluctuations in the size of the surplus.

The findings of this paper suggest that both forces should be present in any successful model of the labor market, both because they are consistent with empirical evidence, and because they have a strong impact on labor market fluctuations. At the same time, the model of this paper is able to generate a rich set of potential labor market responses. Thus, it has the potential to account for the differences between labor market behaviors in different sectors across the U.S. economy and for the differences between labor markets in developed countries around the world.

The empirical results of this paper indicate that the development of increasingly complicated models of the labor market is warranted only if it can improve our understanding of both established as well as some new facts concerning labor market fluctuations. Recent research has been seeking a better understanding of the impact of labor market policies and the efficiency of labor market fluctuations in the context of existing models. By accounting in a simple way for the bulk of labor market fluctuations the model described in this paper provides a benchmark for such analysis.

A large fraction of the literature on labor market fluctuations has focused on a restricted set of moments of the data.[2] This approach often does not capture all of the complexities of labor market behavior. For instance, Lubik (2009) shows that, although the calibration of Shimer's model by Hagedorn and Manovskii (2008) reproduces a set of moments of the data, it falls short of accounting for its dynamic behavior. In order to allow for a rigorous comparison of different specifications of the labor matching model to the data, I estimate them using U.S. data and compare model fit using likelihood methods.

To measure the fit and compare different specifications of the model, I use Bayesian techniques developed for analyzing DSGE models. I also compute the fraction of variations in the data that the model can explain under the best parameter combination. Posterior densities of parameters give a particularly clear picture of how well they are identified and, hence, how important they are for the propagation mechanism.

[2]See among others Hagedorn and Manovskii (2008); Veracierto (2009).

My choice of proper but relatively wide priors guarantees that the results are not sensitive to prior choices.

Both the closed-form solution and the estimation strategy highlight that the explanatory power of the model does not rely on a particular calibration, like that of Hagedorn and Manovskii (2008). They need implausibly small job creation costs to generate large fluctuations in unemployment and vacancies, and a tiny value of the bargaining power of the worker to explain the behavior of real wages. The model developed in this paper fits the data well for a wide range of values of these two parameters because it does not rely on amplifying the response of match surplus to negative shocks.

The main result of the paper is not only robust to alternative parameterizations, but also to alternative data sources. If I use a shorter data set on worker flows constructed from the household survey by Fujita and Ramey (2006), instead of data on job flows constructed from the establishment survey by Davis, Faberman, and Haltiwanger (2006), the fit of the model further improves. This finding reinforces my main results.

This paper contributes to an extensive literature which studies the ability of the labor matching model to explain business cycle facts.[3] The most recent advances in this direction have focused on introducing rich sources of heterogeneity, and their interaction with the labor-leisure trade-off, with capital accumulation and with intra-firm decisions in multi-worker firms in the context of the labor matching model.[4] This paper highlights the two forces considered only partially in this recent work which have the strongest effect on the Beveridge curve and allow the model to capture the bulk of fluctuations in labor market variables.

In this paper, I argue that Shimer's result relies on two crucial simplifying assumptions. First, that the rate at which jobs are destroyed is constant over time. Second,

[3]This literature includes among others Merz (1995); Andolfatto (1996); Cole and Rogerson (1999); Costain and Reiter (2003); Cheron and Langot (2004); den Haan, Ramey, and Watson (2000); Fujita and Ramey (2012).

[4]See Nakajima (2010); Cooper, Haltiwanger, and Willis (2007); Elsby and Michaels (2008); Fujita and Nakajima (2009); Veracierto (2009); Rudanko (2011).

that the cost of job creation varies proportionally with market tightness. Both of these assumptions are inconsistent with empirical evidence. First, Davis, Faberman, and Haltiwanger (2006) have documented that job destruction rates vary substantially over the business cycle. Second, fixed costs of job creation, such as training costs, are a major component of job creation costs. According to Silva and Toledo (2009), 93 percent of costs associated with job creation are incurred after a worker has been hired. This evidence suggests that there is no a priori reason to restrict theoretical analysis to the specification of linear costs of posting vacancies as is common in the literature.[5]

Each of the two key elements of my model has been studied previously, albeit separately. Fujita and Ramey (2012) have studied the effect of endogenous job destruction on the performance of the search and matching model and concluded that alone this element cannot solve the Shimer puzzle. Pissarides (2009) has argued that the effect of introducing training costs into a model with a constant job destruction rate and homogeneous jobs and workers is to reduce the average size of match surplus and to amplify unemployment fluctuations similarly to the calibration of Hagedorn and Manovskii (2008). However, Fujita and Ramey (2012) note that combining endogenous job destruction with the calibration of Hagedorn and Manovskii (2008) does not fully resolve the puzzle.

The model of this paper echoes the findings of den Haan, Ramey, and Watson (2000). Compared to them, the model does not rely on particular assumptions about the allocation of capital and capital adjustment costs. To make the model easier to solve and to allow for rigorous evaluation of model fit, I make simplifying assumptions concerning the evolution of the distribution of match qualities. The mechanics behind the evolution of match qualities in my model are identical to those exhibited by vintage models of creative destruction such as, for instance, Caballero and Hammour (1994). I assume that when the worst jobs are wiped out, the remaining jobs are better on average. This assumption generates a non-persistent response of job

[5]In fact Mortensen and Pissarides (1998) introduced fixed costs into their framework and Pissarides (2009) emphasized the importance of these costs for understanding labor market fluctuations.

destruction consistent with the data. The way the job destruction margin operates in my model also differentiates it from the model of Silva and Toledo (2009) which features a persistent response of job destruction rates to aggregate shocks and training costs.

The model I construct deliberately omits several important aspects of the labor market discussed in the literature. In the benchmark specification, I abstract from on-the-job search and job-to-job transitions, which account for a non-negligible fraction of worker flows.[6] The assumption about the nature of heterogeneity that simplifies the solution of the model also makes employed workers reluctant to search for new jobs. Closer to the end of the paper, I consider an extension of the model with persistent idiosyncratic shocks that can simultaneously account for pro-cyclical fluctuations in job-to-job flows.

The benchmark model abstracts from the interaction between job destruction and capital adjustment as in den Haan, Ramey, and Watson (2000). Introducing capital and incorporating additional vintage effects would significantly complicate the analysis. At the same time, Nakajima (2010) shows that in a rich environment with heterogeneity and borrowing constraints the introduction of capital has a very limited effect of labor market dynamics. Both of these extensions of the model would make the evaluation of model fit computationally infeasible, so I leave them for future work.

The model considered in this paper also abstracts from the multi-worker nature of a firm, wage rigidities, collective bargaining and market power, variations in search effort, labor force participation and many other factors discussed in the literature.[7]

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[6]See Pissarides (1994); Nagypal (2008).

[7]Mortensen and Nagypal (2007) provide an excellent early analysis of the mechanisms at work in the MP model and a thorough overview of its extensions showing that in isolation they do not solve the Shimer puzzle. Introduction of capital and additional vintage effects are studied by Caballero and Hammour (1996); Hornstein, Krusell, and Violante (2005); Eyigungor (2010). Consequences of rigid wages are the focus of Hall (2005); Farmer and Hollenhorst (2006)), and Gertler and Trigari (2006); Krause and Lubik (2007) study decisions of firms with multiple workers, Gertler and Trigari (2006); Rotemberg (2006) study effects of collective bargaining and market power, Veracierto (2008)

Nonetheless, the model explains well the response of the U.S. labor market to aggregate shocks. As such it can serve as a useful starting point for further analysis of the effects of various margins of labor market adjustment and for quantitative studies of labor market policies.

The paper is organized as follows. Section II briefly discusses data on labor market flows. Section III lays out the model and derives the slope of the Beveridge curve. Section IV introduces the empirical methodology, and section V discusses the results. In section VI, I consider two extensions of the model which introduce job-to-job transitions into the framework. I also re-estimate the model using data on worker flows instead of job flows. Section VII concludes. The appendix provides full derivations of model dynamics and additional Figures and Tables characterizing the empirical performance of various models considered in the paper.

II. Labor Market Flows

In a seminal paper, Shimer (2005) concludes that separation rates do not vary much over the business cycle. Ever since, it has become standard in the literature to model the rate at which jobs are destroyed in the U.S. economy and rates at which workers transfer from the unemployed state to the employed state as constant.

This modeling approach has become standard practice despite overwhelming evidence against it. Davis, Faberman, and Haltiwanger (2006) have documented that job destruction rates vary substantially more than job creation rates over the business cycle. Fujita and Ramey (2009) have shown that variations in the separation rate play a non-negligible role in unemployment fluctuations. Barnichon (2012) has demonstrated that fluctuations in job destruction are crucial for understanding the asymmetry of unemployment fluctuations in recessions.

Some of the inconsistencies between surveys of workers and surveys of firms and the resulting series for job destruction/separation rates have been partly responsible for Shimer's conclusion. In this section I use data from three publicly available sources to illustrate the finding that job destruction plays a non-negligible role in unemployment

incorporates the labor force participation decision, and Meyer (1990) measures the discouraging effects of unemployment insurance on search effort.

fluctuations. I use three sources of data to document the behavior of job destruction and job creation rates: Job Openings and Labor Turnover Survey (JOLTS), Current Population Survey (CPS), and Business Employment Dynamics (BED). The CPS contains monthly flows of workers between unemployment and employment since 1990. The BED contains quarterly data on gross job gains by opening and expanding establishments, and gross job losses by contracting and closing establishments since 1992. JOLTS measures monthly rates at which workers are hired and separated from jobs, as well as the breakdown of separations into layoffs and quits. While JOLTS data only starts in 2001, it is helpful in distinguishing transitions into unemployment (layoffs) from job-to-job transitions (quits). Therefore, the amount of job destruction can be inferred from JOLTS data by subtracting quits from separations. Similarly, the amount of job creation can be inferred by subtracting quits from hires.

These three sources of data paint a broadly consistent picture of the scale and magnitude of labor market flows in the U.S. in the last 20 years. The scale of flows is illustrated in Figure 1. About 4 percent of employed workers are laid off from their jobs and enter the unemployment pool during a quarter. Additionally, about 7 percent of employed workers quit and transfer to other jobs during the same period of time. About 60 percent of unemployed workers find a job every 26 weeks, i.e. every quarter. Separations, the sum of quits and job destruction, represent 11 percent of the workforce quarterly. Hires, the sum of job-to-job transitions and job creation, equal separations on average. Flows in and out of the labor force are approximately equal, and relatively stable over the cycle.[8] On average, the unemployment pool represents 6 percent of the labor force.

I plot alternative measures of job destruction and job creation (in millions of jobs per quarter) from all three sources in Figure 2. Figure 2 shows that both the magnitude and cyclicality of fluctuations of flows from employment to unemployment in the U.S. economy are consistent across data sources. Namely, all measures of job destruction increase dramatically at onsets of recessions before returning back to the

[8]For this reason, I abstract from modeling the size of the labor force and fix it at 1 in the theoretical part of the paper.

mean during the recovery phase. The behavior of job creation in recessions is a little less correlated with recessionary episodes across data sources. While some measures show mild increases, others show mild decreases. The observation that is broadly consistent across all data sources is that the response of job creation to a recessionary shock is usually much smaller than that of job destruction. Figure 2 also indicates that there has been some convergence in the behavior of the data from these different sources in the past decade, since the creation of JOLTS.

As shown in Figure 2, at the onset of a recession, a larger than usual number of jobs is destroyed, and workers are laid off. This leads to sharp increases in the numbers of unemployed and dramatic increases in both the number of vacant jobs and job-to-job transitions (as indicated by data from BLS, Conference Board and JOLTS). As the response of job creation is relatively mild (Figure 2), recessions are followed by prolonged periods of high unemployment.

While long time-series on unemployment and vacancies are readily available, the only available source of data on job creation and job destruction, consistent with patterns described in Figure 2 and going back further than 1990, is the data on job creation and job destruction in manufacturing constructed by Davis, Faberman, and Haltiwanger (2006). Since this series is almost perfectly correlated with the series for the whole economy for the available data period, I use a re-scaled version of this data as a proxy for the whole economy for the period starting in 1951 in the empirical analysis.

Reliable data on the behavior of quits from JOLTS only starts from 2001. Partly for this reason, and partly because job-to-job transitions do not directly affect fluctuations in unemployment (workers are effectively exchanging jobs), I abstract from job-to-job transitions in the main body of the paper. In an extension considered in section VI, I show that the predictions of the model allowing for job-to-job transitions are consistent with observed fluctuations in the number of quits in the U.S. economy for the last decade.

In addition to these three sources of estimates, Shimer (2007) and Fujita and Ramey (2006) have constructed indirect measures of job finding and separation rates

FIGURE 1. Labor Market Flows, Quarterly Values.

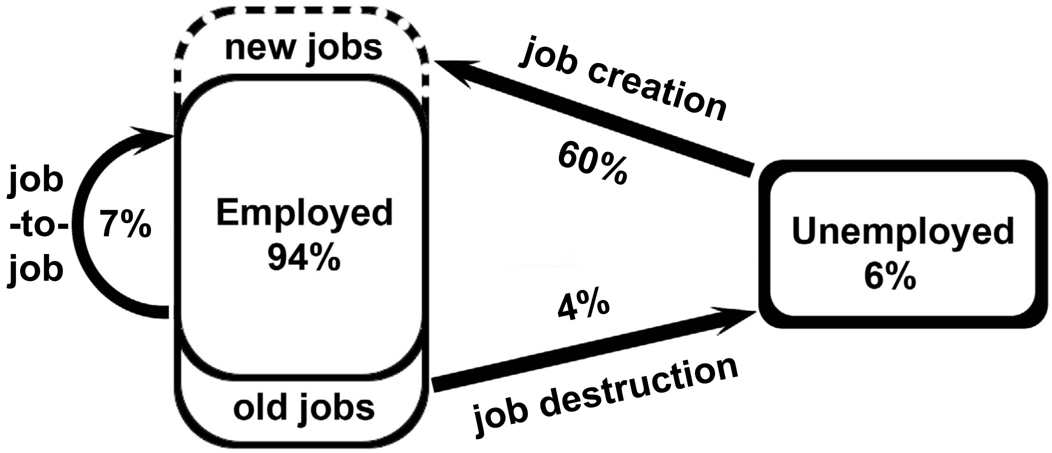
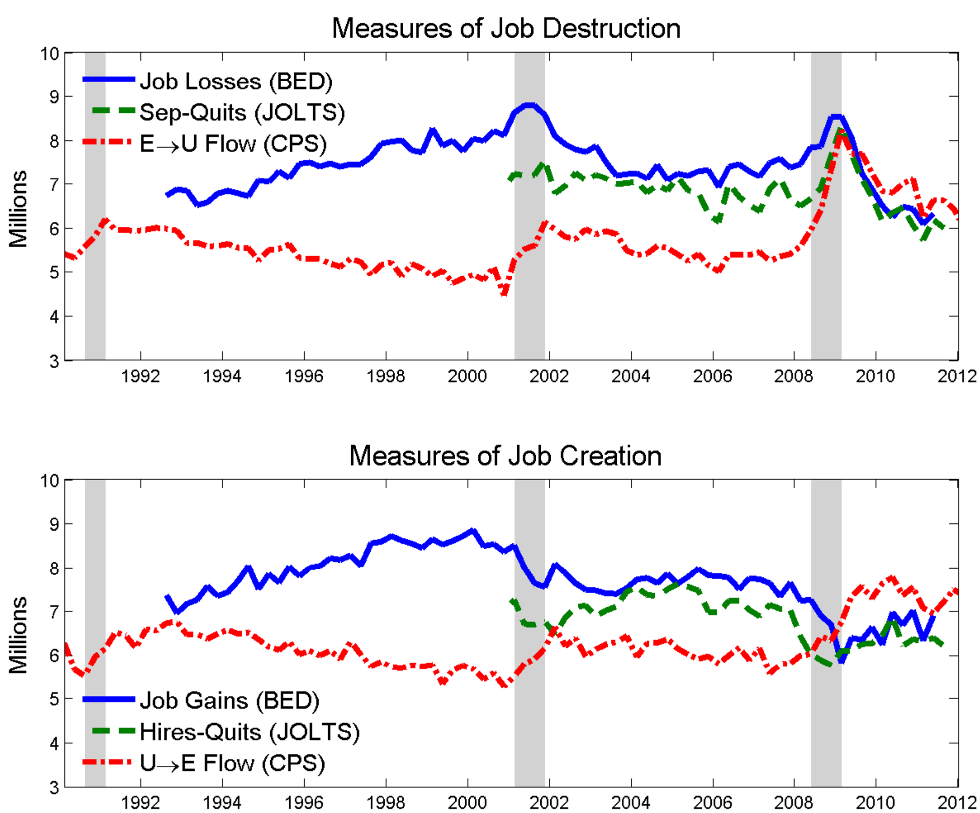


FIGURE 2. Alternative Measures of Job Creation and Job Destruction.



for longer time periods using microeconomic data from the CPS. As an additional robustness check, instead of data on job flows from the establishment survey I use data on worker flows constructed from the household survey by Fujita and Ramey (2006). Although this data is compiled using a few additional assumptions and only goes back to 1976, it reinforces the main findings of this paper. Thus, the main results are robust to the choice of data sources on worker and job flows.

III. Model

Before describing the primitives of the model, I provide an explanation for some of the modeling choices I make. In the Mortensen-Pissarides framework, at every point in time, each job is characterized by an individual productivity level. Differences in productivity lead to differences in profits and wages across jobs. A large enough decrease in the productivity of a job leads to termination of the job at the mutual agreement of the worker and the firm. In this model, aggregate shocks have a non-trivial effect on the productivity distribution, which becomes a state variable. Variations in the number of jobs destroyed are a result of shifts in the productivity distribution over time.

Instead of carrying the productivity distribution, I choose to model the job destruction margin using a specific simplifying assumption. I assume that in every period, the idiosyncratic component of productivity is drawn independently from a distribution with varying support. The size of the support is equal to the number of existing jobs. This makes the number of jobs a state variable charactering the productivity distribution. I use variations in the support of the distribution to capture the idea that once the relatively unproductive jobs are destroyed, the remaining jobs are better on average. This assumption ensures that a persistent aggregate productivity shock does not lead to a persistent increase in the rate of job destruction. The implied dynamics of the distribution of match values are equivalent to those produced by vintage models of creative destruction such as Caballero and Hammour (1994). The firm-specific shocks to consumer tastes in my model are exactly equivalent to idiosyncratic productivity shocks. This specification is chosen solely for the purpose of transparency: to distinguish idiosyncratic shocks from aggregate shocks.

The model I construct is a real business cycle model with a matching friction. I deliberately simplify the model to concentrate the discussion around the two key elements: endogenous job destruction and training costs. First, I describe the physical environment. Then I explain how employment relationships between workers and firms are formed, operated and terminated. I close the model with a description of the household's problem and equilibrium conditions. I then explain how incorporating endogenous job destruction and training costs affects the propagation of shocks.

III.1. **Physical Environment.** Time is discrete and continues forever. The economy is populated by a unit measure of workers and a large number of firms. Workers can be unemployed searching for a job or engaged in a productive employment relationship. I denote the measure of unemployed, U_t, and N_t represents the measure of workers engaged in productive activities. Their sum is equal to the total number of workers:

$$N_t + U_t = 1. \tag{1}$$

Each firm has a blueprint for producing a variety of intermediate good and needs a worker to be productive. A firm can be in one of three states: matched with a worker and producing, searching for a worker or idle. A firm can hire at most one worker, who provides at most one unit of time. As operating firms always demand the maximum amount of time, N_t represents both the measure of workers in productive activities and the measure of operating firms. I denote the measure of firms searching for a worker V_t, which also represents the number of vacancies. The measure of idle firms is sufficiently large so there are always enough potential entrants.

The production technology of a firm is linear in labor so that each worker produces A_t units of the intermediate good. A_t represents aggregate labor productivity and follows an autoregressive process of order one governed by exogenous productivity shocks, ε_t, drawn i.i.d. from a standard normal distribution:

$$A_t = A_{ss}^{1-\rho_A} A_{t-1}^{\rho_A} e^{\sigma_A \varepsilon_t}, \quad \varepsilon_t \in \mathcal{N}\left(0,1\right), \tag{2}$$

where A_{ss} is the steady-state value of productivity, ρ_A is persistence, and σ_A is the standard deviation of shocks to labor productivity.

New employment relationships are formed through a matching process between firms with openings and unemployed workers. The mass V_t of firms that decide to post vacancies is matched with the mass of unemployed workers U_t according to a constant returns to scale matching function[9]:

$$M_t = BU_t^{\alpha}V_t^{1-\alpha}, \tag{3}$$

where M_t is the mass of new employment relationships starting to operate in the next period, and α is the matching elasticity.

The cost of job creation has two components: a *recruiting* component includes costs of advertising and interviewing, and a *training* component includes costs of setting up a working environment and training a worker to meet specific needs. Thus, firms post vacancies at a cost c and then firms matched with workers incur an additional training cost K per match. The total cost of job creation in units of consumption, X_t, satisfies:

$$X_t = cV_t + KM_t. \tag{4}$$

I assume that workers are members of a large family that pools income and then distributes it equally to all members. Then, the household maximizes the expected discounted utility of a representative worker, which values consumption and leisure. The final consumption good is aggregated from different varieties of intermediate goods produced by firms:

$$Y_t = \int_0^{N_t} z_{it}q_{it}di, \tag{5}$$

where z_{it} denotes the idiosyncratic taste shock for variety of firm i, q_{it} is the input of variety i, and N_t is the measure of productive units operating in period t. I assume that the taste shock is drawn from a distribution with variable support:

[9]Given the parameter values, the condition $M_t \leq min\left(U_t, V_t\right)$ holds in all the simulations.

$$z_{it} = e^{-gi}, \qquad i \in \mathcal{U}\left[0, J_t\right], \tag{6}$$

where g is the slope parameter of the distribution, i indexes firms uniformly distributed on a closed interval $[0, J_t]$, and J_t is the measure of jobs available at the beginning of period t. Assuming the final good to be the numeraire, the price of intermediate good i, p_{it}, is then directly pinned down by the taste shock:

$$p_{it} = z_{it}. \tag{7}$$

At the beginning of each period, after aggregate productivity and tastes become known, firms and workers in existing productive relationships meet and decide whether to preserve the relationship or terminate it. I follow the literature by assuming that if they decide to keep it, they split the surplus using a Nash bargaining solution. I denote Ψ the bargaining power of a worker. The threat point of the worker is to become unemployed and the threat point of the firm is to become idle. Firms and workers discount the future at the same rate.

I denote ζ_t the fraction of jobs that are terminated at mutual agreement of the worker and the firm. Workers join the unemployment pool and start searching for new jobs during the same period. The number of productive units that keep operating in period t is:

$$N_t = J_t\left(1 - \zeta_t\right). \tag{8}$$

While N_t units produce intermediate goods, idle firms open V_t new positions and hire M_t unemployed workers to fill them. These workers are trained in period t to become productive in period $t+1$. The training cost, K, is split between the worker and the firm in the same proportion as their future surpluses.

The number of jobs carried to the next period is the sum of survivors at the end of period t, N_t, and new matches, M_t:

$$J_{t+1} = N_t + M_t. \tag{9}$$

Having described the primitives, technologies and preferences, I now describe the competitive equilibrium in this economy.

III.2. **Characterization of Equilibrium.** First, I describe the household's problem. The solution of this problem determines the consumption-leisure trade-off of the household. Second, I derive the continuation values of firms and workers, and describe how they split the total surplus of the match through bargaining. Third, I discuss the problem firm i and worker i face, when deciding whether to terminate their relationship. Finally, I describe how idle firms choose their recruiting activity. I conclude by defining a competitive equilibrium.

The representative household chooses consumption and labor input to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right),$$

subject to the budget constraint:

$$C_t = bU_t + \int_0^{N_t} W_{it} di + \Pi_t - \Psi K M_t,$$

where W_{it} is the wage paid to worker i in period t, and Π_t is the sum of firm profits. All of wage and profit income net of training costs borne by the workers is spent on consumption, C_t, of final goods produced in the same period. Unemployed workers are assumed to receive benefits, b.[10] In equilibrium, markets for all varieties of intermediate goods clear:

$$q_{it} = A_t. \tag{10}$$

Therefore, aggregate profits, Π_t, are the sum of individual profits of firms net of job creation costs:

$$\Pi_t = \int_0^{N_t} \left(p_{it} A_t - W_{it}\right) di - cV_t - \left(1 - \Psi\right) K M_t.$$

[10]The unemployment benefit can be thought of as the sum of unemployment insurance payments and the value of home production net of the costs of search.

Together, these conditions imply the resource constraint, which allocates output of the final goods coming from market activities and home production to consumption and costs of job creation:

$$C_t + X_t = Y_t + bU_t. \tag{11}$$

I use a standard preference specification which borrows infinite Frisch elasticity of labor supply from the model of indivisible labor and employment lotteries of Hansen (1985) and Rogerson (1988)[11]:

$$u\left(C_t, N_t\right) = \ln C_t - \varpi N_t.$$

where parameter ϖ captures the marginal disutility of labor. The problem of the representative household can be expressed as a problem of maximizing a Lagrangian function. Then, the Lagrange multiplier on the budget constraint, λ_t, must satisfy:

$$\lambda_t = \frac{1}{C_t}. \tag{12}$$

Maximization over N_t determines the outside option of the worker. When adding a worker to the employment pool, the household faces the prospect of giving away the value of worker being unemployed, denoted w_t, which encompasses the unemployment benefit, b, the value of leisure, and the option value of finding a new job in the future:

$$w_t = b + \frac{\varpi}{\lambda_t} + \frac{M_t}{U_t}\left(\Gamma_t^W - \Psi K\right). \tag{13}$$

In the equation above, the ratio of matches to unemployment, $\frac{M_t}{U_t}$, is the probability of finding a job, and Γ_t^W is the worker's expected future benefit from engaging in an employment relationship. The future benefit is taken net of the training cost, which is split between the worker and the firm.

The total value of job i to the worker is the present discounted sum of wages, W_{it}, net of her outside option, w_t. When deciding whether to preserve the relationship

[11] In the Appendix VIII.2 I consider a much more general specification of utility with alternative values of risk aversion and labor supply elasticity. Because both of these parameters are not identified separately from other parameters of the model, I fix them at these conventional values.

with the firm, the worker compares this total benefit to the alternative of walking away and getting his outside option. Therefore, the value of job i to the worker, V_{it}^{W}, satisfies:

$$V_{it}^{W} = \max\left\{W_{it} - w_t + E_{it}\beta\frac{\lambda_{t+1}}{\lambda_t}V_{i't}^{W}, 0\right\}.$$

where i' denotes the index of the firm in the next period. Because of the simplifying assumption that taste shocks are i.i.d., the values of future benefits to the worker, Γ_t^W, are independent of i:

$$\Gamma_t^W = E_{it}\beta\frac{\lambda_{t+1}}{\lambda_t}V_{i't+1}^{W} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\max\left\{W_{i't+1} - w_{t+1} + \Gamma_{t+1}^W, 0\right\}. \tag{14}$$

Similarly, the present discounted sum of profits of firm i is compared to the alternative of walking away and getting nothing. The value of the job to firm i, V_{it}^F, satisfies:

$$V_{it}^{F} = \max\left\{p_{it}A_t - W_{it} + E_{it}\beta\frac{\lambda_{t+1}}{\lambda_t}V_{i't}^{F}, 0\right\}.$$

Likewise, the values of future benefits to firms, Γ_t^F, are all equal:

$$\Gamma_t^F = E_{it}\beta\frac{\lambda_{t+1}}{\lambda_t}V_{i't+1}^{F} = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\max\left\{p_{i't+1}A_{t+1} - W_{i't+1} + \Gamma_{t+1}^F, 0\right\}. \tag{15}$$

Every period the firm and the worker bargain over the wage, W_{it}, which splits the current surplus in fixed proportions:

$$W_{it} - w_t = \Psi\left(p_{it}A_t - w_t\right). \tag{16}$$

From combining this equation with equations (14) and (15) above, it follows that future and total surpluses are split in the same proportions:

$$\Gamma_t^W = \Psi\left(\Gamma_t^W + \Gamma_t^F\right) = \Psi\Gamma_t,$$

where joint future surplus, Γ_t, is defined as follows:

$$\Gamma_t = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \max \left\{ p_{i't+1} A_{t+1} - w_{t+1} + \Gamma_{t+1}, 0 \right\}. \tag{17}$$

Since taste shocks (6) are strictly decreasing in i by construction, and prices (7) are linear in tastes; profits, wages and match values are all strictly decreasing in i. Wage bargaining condition (16) and surplus split (17) together imply that the value of a job to the firm and to the worker equal zero simultaneously. Hence, there exists a unique cutoff value i^*, such that worker i^* and firm i^* are indifferent between terminating their relationship and keeping it. For all $i > i^*$ the worker and the firm mutually agree to terminate their relationship. For all $i \leq i^*$ the worker and the firm prefer to keep it. The cutoff, $i^* = N_t$, satisfies:

$$p_{it} A_t|_{i=N_t} - w_t + \Gamma_t = 0. \tag{18}$$

This equation determines the number of surviving jobs, N_t, the cutoff price, $\underline{p}_t = p_{N_t,t}$, and the efficient endogenous rate of job destruction, ζ_t. Firms and workers terminate their relationships when the sum of current and future surpluses becomes negative.

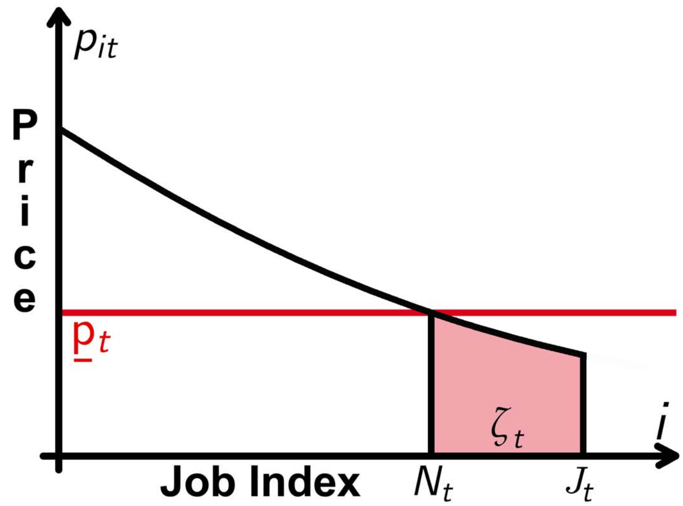
Finally, free entry of new firms into the labor market guarantees that vacancies are open until their expected marginal costs are equal to their expected marginal benefits:

$$c = \frac{M_t}{V_t} \left(\Gamma_t^F - (1 - \Psi) K \right), \tag{19}$$

where $\frac{M_t}{V_t}$ is the vacancy filling rate, which firms take as given. Thus, the number of firms advertising vacancies is such that the cost of posting an extra vacancy equals the expected future benefit of a match, net the cost of training the worker if the vacancy is filled.

A competitive equilibrium of the model economy is a solution to equations (1)-(19), where $\{U_t; A_t; M_t; X_t;\ Y_t; z_{it}; p_{it};\ \zeta_t; J_t;\ q_{it}; p_{it};\ C_t; \lambda_t; w_t;\ \Gamma_t^W; \Gamma_t^F;\ W_{it}; \Gamma_t; N_t; V_t\}$ are endogenous variables and ε_t is the exogenous shock.

FIGURE 3. Price Distribution and the Cutoff Price.



III.3. **Propagation Mechanism.** In this subsection, I first make some general observations about the properties of the model. Then I describe the two key elements of the model, and how they jointly determine the response of unemployment and vacancies to aggregate shocks. Finally, I derive the slope of the Beveridge curve and show how it is affected by the parameters of the model.

First, notice that shocks to aggregate demand could be introduced into the model by having an aggregate component of taste for consumption, Z_t. Demand shocks would enter linearly into the price, p_{it}, and, therefore, would be indistinguishable from supply shocks, A_t. Thus, the model describes the response of the labor market to aggregate shocks, which could come from both the demand and the supply sides.

Second, compared to the MP model, this is a general equilibrium model. The advantage of general equilibrium analysis is that it can simultaneously take into account variations in the outside option of the worker due to the consumption-leisure trade-off, as well as unemployment benefits, search costs and other factors. At the same time it decouples parameters that determine the size of variations in the value of a match from parameters that affect average match value. The first set of parameters relates variations in the value of the match to the marginal utility of consumption of a representative household. The second set of parameters pins down the average match value by equalizing to average costs of job creation over the lifetime of a job.

Now I move on to the discussion of how the two key elements of the model work. The job destruction margin is the first key element of the model. Figure 3 depicts the price distribution as a function of the job index, $i \in [0, J_t]$. The cutoff price level, $\underline{p}_t$, corresponds to the number of productive jobs, N_t. A fraction ζ_t of available jobs that are not worth operating according to equation (18) are terminated at the mutual agreement of the worker and the firm. In steady-state, all of the destroyed jobs are replaced by new matches.

A negative productivity shock leads to a persistent decrease in productivity, A_t, and results in a decrease in expected future benefits, Γ_t. This shifts the cutoff price upward and leads to a spike in job destruction and a consequent increase in unemployment.

To illustrate the effects of structural parameters on the response of the value of a match, unemployment and vacancies to variations in productivity, I linearize equations (1)-(19) around the steady-state and substitute them into each other to obtain the elasticity of the value of the match, unemployment and vacancies with respect to productivity[12]:

$$\frac{\hat{U}_t}{\hat{A}_t} = -\frac{1 - U_{ss}}{U_{ss}}\Lambda, \tag{20}$$

$$\frac{\hat{\Gamma}_t}{\hat{A}_t} = \tau, \tag{21}$$

where U_{ss} is the steady-state unemployment rate, and τ and Λ are the elasticities of the average match surplus and of employment to productivity shocks. Both of these elasticities take on values in the interval $[0.5, 2]$ for all reasonable values of parameters. This fact demonstrates that the propagation mechanism of the model does not involve amplification of fluctuations in the value of the match as done by the calibration of Hagedorn and Manovskii (2008). Instead, it builds on the ability of the endogenous job destruction margin to amplify unemployment fluctuations and on the interaction of this margin with training costs to attenuate fluctuations in labor demand.

[12] Full derivation of the steady-state and the linearized equations are described in Appendix VIII.1.

Training costs are the second key element of the model. They help explain the response of vacancies and job creation to productivity shocks. To demonstrate the effect of training costs, I linearize equation (19) and substitute in the matching function (3) to obtain:

$$\hat{U}_t - \hat{V}_t = \frac{\hat{\Gamma}_t}{\alpha\left(1-\varphi\right)}, \tag{22}$$

where α is the elasticity of the matching function, $\varphi = \frac{\varphi_2(1-\Psi)}{\varphi_2(1-\Psi)+1-\varphi_2}$ is the fraction of training costs incurred by the firm in proportion to total costs incurred by the firm, and $\varphi_2 = \frac{KM_{ss}}{cV_{ss}+KM_{ss}}$ is the fraction of training costs in total costs of job creation. This equation shows how training costs modulate the response of labor market tightness to variations in the value of a match. When a large number of jobs are destroyed in response to a negative aggregate shock, and when the value of the match does not respond much, market tightness has to change dramatically to be consistent with a drop in the vacancy rate observed in the data.

When training costs are absent, $\varphi \to 0$, the response of market tightness to changes in prospects of future profits is small. A negative productivity shock leads to a sharp increase in unemployment, which through a mild response in market tightness leads to an increase in the vacancy rate. Thus, when most of the costs are recruiting costs, a sharp increase in unemployment makes workers much easier to find, encouraging firms to post more vacancies.

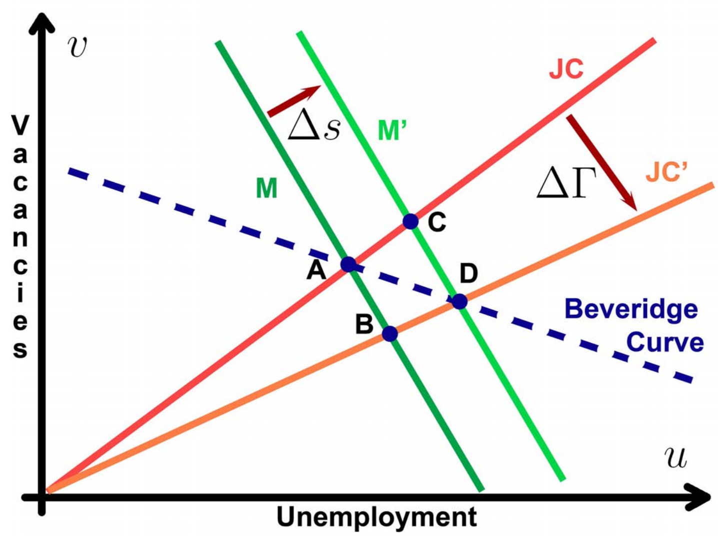
On the contrary, when most of the costs are training costs, $\varphi \to 1$, even a mild decrease in the value of the match leads to a sharp decrease in market tightness. Thus, training costs attenuate the response of total costs to market tightness, discouraging firms from opening vacancies in a recession.

Combining equation (22) with equations (21) and (20) I derive the slope of the Beveridge curve:

$$\frac{\hat{V}_t}{\hat{U}_t} = 1 - \frac{1}{\alpha\left(1-\varphi\right)}\frac{U_{ss}}{1-U_{ss}}\frac{\tau}{\Lambda}. \tag{23}$$

The slope of the Beveridge curve is strongly affected not only by the elasticity of the matching function, α, but also by the relative size of training costs, φ. The values

FIGURE 4. Beveridge Curve.



of τ and Λ, defined by equations (20) and (21), are also important for understanding the size of movements along the Beveridge curve, but have only a mild effect on the slope of the curve.

III.4. **Illustrative example.** To illustrate the combined effect of the job destruction margin and training costs, I use a comparative statics exercise. I look at three cases: constant exogenous job destruction, as well as endogenous job destruction with and without training costs.

To give a numerical illustration, I set tentative values for the key parameters. One can infer the elasticity of the matching function directly from comparing the volatilities of market tightness and the job finding rate following Shimer (2005). I use this method to set α to 0.7. I build on evidence from Silva and Toledo (2009) to infer the size of total costs and its split into recruiting and training costs. Using this evidence, I set total job creation costs to 35 percent of the quarterly wage of a typical employee, and φ to match the observation that training costs account for 93 percent of job creation costs incurred by firms. I also set b to 0.4 - the typical value for the size of benefits in the search literature, and U_{ss} to 5.6 percent - the historical average unemployment rate in the U.S. This gives values of Λ at 0.85 and τ at 1.4.[13]

First, when jobs are destroyed at an exogenously given rate, the Beveridge curve coincides with an isoquant of the matching function. Its slope is determined exclusively by the elasticity of the matching function, α:

$$\frac{\hat{V}_t}{\hat{U}_t} = \frac{-\alpha}{1-\alpha} = -2.33. \tag{I}$$

Second, when firms are allowed to choose whether to destroy jobs based on future profits, and all job creation costs are recruiting costs, the Beveridge curve is positively sloped:

$$\frac{\hat{V}_t}{\hat{U}_t} = 1 - \frac{1}{\alpha}\frac{U_{ss}}{1-U_{ss}}\frac{\tau}{\Lambda} = 0.87. \tag{II}$$

Increasing the fraction allocated to training costs solves this problem. When training costs are set to correspond to 93 percent of job creation costs, leaving 7 percent to recruiting costs, the predicted slope of the Beveridge curve comes close to the slope of -1 - the slope of U.S. unemployment and vacancy data.

$$\frac{\hat{V}_t}{\hat{U}_t} = 1 - \frac{1}{\alpha\,(1-\varphi)}\frac{U_{ss}}{1-U_{ss}}\frac{\tau}{\Lambda} = -0.91. \tag{III}$$

Figure 4 illustrates in the unemployment-vacancy space how the slope of the Beveridge curve is determined by a combination of shifts in the matching curve and the job creation curve. Let point A be the original steady-state. In case (I), when job destruction is given exogenously, variations in the value of the match shift the job creation curve (22) clockwise. The economy moves along the isoquant of the matching function (3) to point B. In case (II), when job destruction is endogenous, but training costs are absent, a spike in job destruction shifts the matching curve upwards. This shift is much larger than the shift in the job creation curve, so the economy ends up in point C. Finally, when training costs are a large fraction of total

[13]Specific values for benefits and total search costs are not very important for the outcome of the illustrative example. Depending on these parameters, elasticities τ and Λ take values in the interval $[0.5, 2]$, and their ratio belongs to the interval $[1, 2]$.

FIGURE 5. Impulse Responses to a 1% Negative Productivity Shock.

costs, an aggregate shock leads to a concerted movement in both the matching curve and the job creation curve, leading the economy to point D. This corresponds to case (III), with the slope of the Beveridge curve resembling the slope observed in the data.

The dynamic response of the calibrated model is summarized by impulse response functions to a productivity shock depicted in Figure 5. It works as follows: A negative productivity shock lowers contemporaneous profits of firms leading to a sharp increase in job destruction. As more workers lose their jobs the number of unemployed workers increases, loosening the labor market. A decline in contemporaneous productivity also leads to a decline in expected future profits. This lowers the benefits to firms of creating new jobs and, because of the mild response of job creation costs, significantly undermines their incentives to open vacancies. The number of vacancies falls. As the number of employment opportunities shrinks due to lower productivity, the number of newly created jobs does not respond much.

After a sharp employment adjustment in the first period, the least productive jobs have already been destroyed and the job destruction rate quickly returns close to its original level. As productivity slowly recovers, the cutoff price for job destruction slowly returns to its original level. As firms see an increase in future profits, they start opening more vacancies and creating more jobs.[14]

IV. Empirical Methodology

To explore the ability of the model to fit the data I use Bayesian methods for analyzing DSGE models.[15] This methodology has several advantages when compared to commonly used calibration strategies. In the context of vigorous debates over parameters of the standard matching model, the Bayesian framework allows me to remain agnostic. I let the data choose a calibration that is most likely to explain its behavior.

The second advantage of this methodology is that a likelihood function gives natural weights to different moments of the data instead of focusing on just a few. In addition, setting relatively diffuse priors allows me to conduct a sensitivity analysis of model performance to the parameter combination. If I find that a posterior estimate is as wide as the prior, then the exact value of the corresponding parameter is not important for explaining the data. Conversely, a narrow posterior estimate means that model dynamics are very sensitive to the exact calibration of that parameter.

In this section, I describe the strategy that I use to evaluate the model. First, I solve for the steady-state of the model. I then log-linearize the equations of the model around the steady-state and solve the resulting system of linear forward-looking equations using a method developed by Sims (2002). This gives me the state-space representation of the model:

[14]Depending on the share of training costs, the response of job creation to aggregate shocks can take both positive and negative values. The positive response of unemployment could be smaller or the negative response of vacancies could be bigger. As a result, the equilibrium number of matches determined by matching function (3) could respond negatively.

[15]A survey of these methods is provided for instance by An and Schorfheide (2007).

$$X_t = FX_{t-1} + G\varepsilon_t \tag{24}$$

$$Y_t = HX_t + \upsilon_t, \tag{25}$$

where X_t is the vector of state variables, Y_t is the vector of observables, and F,G and H are matrices. I assume that the innovation to labor productivity, ε_t, is the only exogenous shock in the model. I attribute all the residual variation in observed fluctuations to a vector of measurement errors, υ_t. The fraction of variations in Y_t explained by the model is represented by HX_t and the unexplained component is captured by the error term. To allow for enough variation in the data and to avoid stochastic singularity, I assume there are as many sources of measurement error as there are observables so that each measurement equation has its own error term.[16]

I treat the model as the data-generating process and use the Kalman filter to construct the likelihood function of the data conditional on parameters. I combine the likelihood function with the prior distribution of parameters to obtain the posterior distribution of parameters and use the random-walk Metropolis-Hastings algorithm to explore it numerically.[17] I then use the Kalman filter to obtain smoothed estimates of the shock process for labor productivity using parameter values at posterior mode.

IV.1. **Data.** For estimation, I use seven observables: unemployment, vacancies, job destruction, job creation, and job finding rates, real wages and labor productivity. All data are quarterly, seasonally adjusted for the period 1951:1 - 2010:4.[18] The

[16]To avoid stochastic singularity I need at least as many shocks as observed variables. If I include productivity shocks, I can exclude one of the measurement errors. I choose not to do so because that would imply a prior choice of the variable I want the model to explain exactly. I choose to remain agnostic about the choice of variables the model can best explain by setting wide priors on standard deviations of measurement errors.

[17]The algorithm is extensively discussed in Geweke (1999). I use the open source DYNARE software developed by Collard and Juillard (2003) and collaborators.

[18]To avoid merging data series from different sources for job creation, job destruction, job finding rate and vacancies, I could restrict selection to a time interval ending in 2004:4 and exclude the recession of 2008. This would not change any of my conclusions, so I prefer to include the most recent recession episode.

unemployment series is the unemployment rate for those older than age 16, provided by the BLS. The vacancy series is the index of help-wanted advertisements provided by the Conference Board before 2001, merged with JOLTS data after 2001. The series for real wages is constructed by dividing average hourly earnings in private nonfarm payrolls by the consumption price index.

As a proxy for job destruction and job creation, I use destruction and creation rates in manufacturing constructed by Davis, Faberman, and Haltiwanger (2006). Davis, Faberman, and Haltiwanger (2006) also provide series for all sectors for a much shorter period of time. The series for manufacturing and for all sectors have notably different volatilities, but a correlation close to one.[19] I use this observation to scale the series for manufacturing to represent the whole economy. For the period after 2005, I augment the series with rates of job loss in contracting establishments and job gains in expanding establishments in manufacturing supplied by BED. Finally, I use the job finding rate series computed from CPS data by Shimer (2005) and augment it for the period after 2007 with the transition rate from unemployment to employment constructed from CPS data. For the robustness exercise I use job finding and separation rates constructed by Fujita and Ramey (2006) for the period 1976:I - 2005:IV to compute the job finding, job creation and job destruction rates.

I use the series for labor productivity, measured as real output per worker in the non-farm business sector. This series is constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. For a consistency check, I also construct data series for profits and output. For the profits series, I take nominal corporate profits before taxes from the BEA and divide them by the nominal value of GDP. The output series is the real GDP index provided by the BEA divided by the labor force. I apply the Hodrick-Prescott filter with smoothing parameter 1600 to detrend all series.

[19] See Figure 9 in the Appendix.

TABLE 1. Prior Distributions

Parameter		Density	Mean	Std. Dev.
Discount factor	β	Fixed	0.99	-
Matching elasticity	α	Beta	0.5	0.2
Bargaining power of worker	Ψ	Beta	0.5	0.2
Unemployment benefit	b	Beta	0.4	0.1
Unemployment rate	u	Fixed	.056	-
Job destruction rate	s	Gamma	0.04	0.01
Fraction of training costs	φ	Beta	0.5	0.25
Total costs	μ	Beta	0.3	0.1
Persistence of productivity	ρ_A	Beta	0.5	0.2
Shock Standard Deviation	σ_A	Beta	0.005	0.0025

IV.2. **Priors.** There are nine structural parameters in the model, of which $\{g, B, c, K\}$ are hard to directly compare with micro estimates. Instead of estimating them directly, I construct an alternative set of steady-state values that I then treat as parameters. I define $u = U_{ss}$ - the steady-state unemployment rate, $s = \zeta_{ss}$ - the job destruction rate, φ - the fraction of training costs in total job creation costs incurred by firms and μ - the sum of recruiting and training costs per employee incurred by a firm as a fraction of their quarterly wage. I then use the fact that conditional on the rest of the parameters, there is a one-to-one mapping between $\{g, B, c, K\}$ and $\{u, s, \varphi, \mu\}$.

Prior distributions are reported in Table 1. I choose prior means based on values used in previous studies. I make the priors uninformative by setting prior standard deviations to relatively large values whenever possible. This allows me to remain agnostic and let the data choose the parameter combination that is most likely to capture the dynamic properties of the data. For parameters with support on the unit interval, I use the Beta distribution and for real-valued parameters I use the Gamma distribution.

I set the discount factor, β, to 0.99. The unemployment rate is fixed at its historical mean of 5.6 percent. Based on evidence from Silva and Toledo (2009), I set the prior on the total cost of job creation to 30 percent of the quarterly wage of a new hire and the prior on the fraction of training costs to 50 percent of the job-creation amount. I allow for large variations in both of these values.

I set the prior on the steady-state job destruction rate at 4 percent to match the average flow from employment to unemployment during a quarter.[20] I choose to be completely agnostic about the bargaining power, the matching elasticity, the curvature of demand and the autoregressive parameter of labor productivity. As priors for standard deviations of errors, I choose inverse-gamma distributions with standard deviations of 0.5 percent for productivity and 2 percent for all other variables. I run 10 blocks of 5000 iterations each from different starting points and target an acceptance rate of 30 percent.

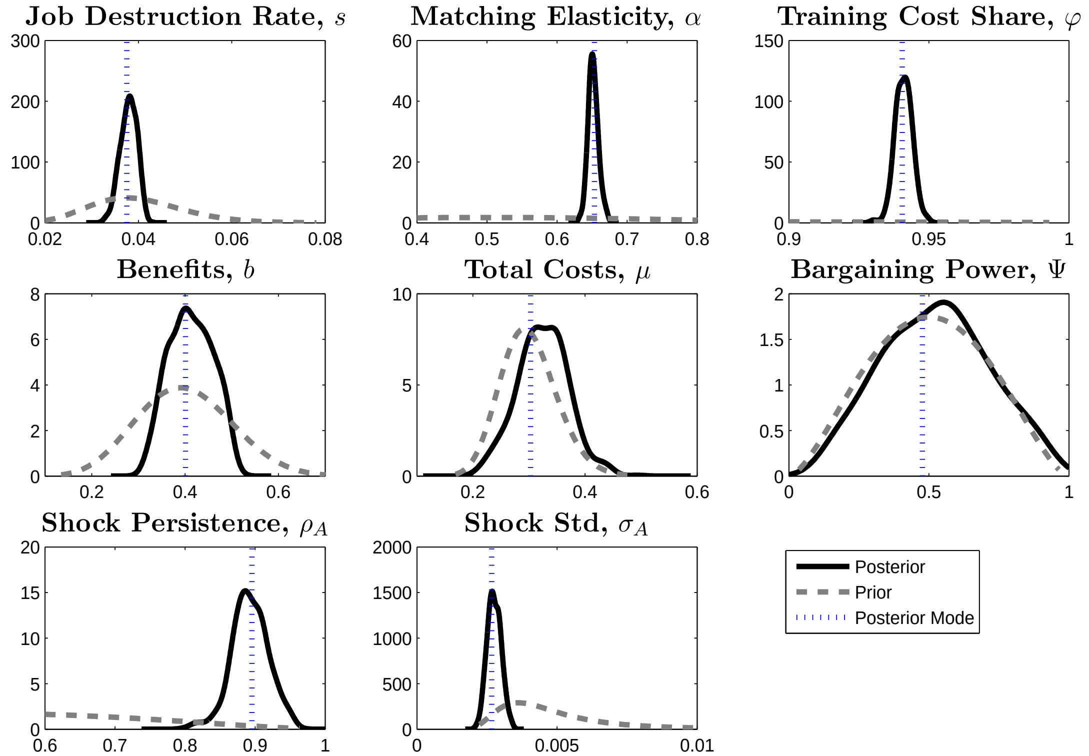
V. Results

In this section I describe the posterior estimates and discuss their implications for calibration of labor matching models. I then evaluate the fit of the model along different dimensions and use values of the marginal density to evaluate the relative importance of the two key elements.

V.1. **Parameter Estimates.** I report distributions of posterior estimates in Figure 19. The posterior mode of the average job destruction rate is estimated between 3 and 4 percent. The posterior estimate is much narrower than the the prior and implies a job finding rate of 50 to 70 percent. This is contrary to the finding of Cole and Rogerson (1999) that a relatively low job finding rate is required to match data on job creation and job destruction implying counterfactually long duration of

[20]For a motivation of this prior see Section II. This value is broadly consistent with the findings of Nagypal (2008) that only about 25 percent of all separations (which are approximately 10 percent per quarter) correspond to transitions from employment to unemployment. Also, according to the distribution of unemployment duration provided by the BLS, about 60 percent of all unemployed find jobs within a quarter, which is about 4 percent of employment in steady-state.

FIGURE 6. Comparison of Prior and Posterior Distributions.



unemployment. Instead, the implied estimate of duration is at the lower bound of plausible duration values.

The estimate of matching elasticity has a very narrow posterior distribution around the mode of 0.65, close to Shimer's estimate of 0.72. This is not surprising given that the parameter is identified in a similar way through the relationship between the job finding rate and market tightness.

The estimate for the size of unemployment benefits, b, covers the interval between 0.3 and 0.5. This is consistent with the value of 0.4 used by Shimer and by studies of the effects of unemployment benefits and wage rigidities, and much lower than the value of 0.95 used in the calibration of Hagedorn and Manovskii. This shows that my results are obtained under a conventional value of this parameter.

The posterior estimate of the bargaining power of workers, Ψ, has a very wide confidence interval: from 26 percent to 90 percent. In fact, the posterior virtually coincides with the prior. This implies that the value of bargaining power has little or no effect on the dynamic properties of the model. This confirms the analytical

TABLE 2. Comparison of Second Moments

Standard Deviations								
	Y	U	V	JD	JC	JF	W	Y/N
Data	1.63	12.8	14.5	12.8	7.8	8.1	0.87	1.09
Model	1.01	9.63	14.4	10.4	3.9	8.3	0.44	0.63
Correlations with GDP								
Data	1	-0.82	0.81	-0.62	0.26	0.78	0.16	0.56
Model	1	-0.99	0.99	-0.51	-0.99	0.99	0.99	0.99

expressions described in equations (20)-(23) – only the way in which training costs are split matters. Since this effect is also accounted for by the parameter φ, the finding that bargaining power does not affect model performance is not at all surprising. The only instance at which the data are directly affected by the bargaining power of the workers is the volatility of real wages. Like labor productivity, the series for real wages has a large measurement error, driven mostly by changes in the consumption price index, while nominal wages remain largely unchanged over the cycle. The model prefers to attribute most of variations in the wage series to measurement error rather than placing significant weight on its random movements.[21]

The estimate for the total job creation cost, μ, incurred by a firm lies in a wide range from 20 to 40 percent of quarterly wages of a new hire. The fraction of training costs and other costs specific to a match in total creation costs, φ, is tightly estimated between 93 and 95 percent. As noted before, the second parameter is key to explaining the behavior of vacancies and the negatively sloped Beveridge curve. Both of these parameters match quite closely the evidence presented by Silva and Toledo (2009). They estimate total costs to be between 36 and 55 percent of the quarterly wage of a new hire, with the fraction of training costs estimated at around 93 percent.

The result that bargaining power and the size of total job creation costs do not play a very important role in explaining the behavior of unemployment, vacancies and wages over the business cycle is in stark contrast with existing theoretical literature.

[21]See Figures 10-12 in the Appendix.

The reason for this is the omission of the job destruction margin. When firms are not allowed to close positions in response to deteriorating aggregate conditions, a much larger decline in the value of a match is required to explain the increase in unemployment through the job creation margin alone. For variations in match value to be large, the costs of job creation have to be tiny, and the corresponding level of unemployment benefits enormous. Similarly, when variations in match value are large, the bargaining power of a worker has to be unreasonably small to match low variability in wages. The introduction of endogenous job destruction into the model eliminates the need for such extreme parametric assumptions. [22]

V.2. **Model Fit.** To evaluate the fit of the model, I compare a selected set of moments of the data with moments of artificial data generated by the model when hit by the estimated productivity shock. Table 2 compares standard deviations of eight observables of interest as well as their correlations with output. The results indicate that the model fits the data well, explaining virtually all of the fluctuations in vacancies, job destruction and job finding rates, three-quarters of fluctuations in unemployment, and half of variations in job creation, with a single aggregate shock. The required variations in labor productivity and implied variations in wages are both of reasonable magnitude. Given the simplicity of the model this is a remarkable result.

The model matches well most of the signs of cross correlations between observables with one exception. In the data job creation responds to recessionary shocks negatively at first and then rebounds as workers are rehired. The model predicts an immediate rebound of job creation. When compared to the model, the data on job creation has a lag of about two quarters. This is essentially the only dimension on which the model doesn't perform well. This inconsistency can be easily solved by introducing an additional assumption that matches created in period t enter the

[22] I do a simple check for consistency of these parameters using their implications for the behavior of profits. Figure 10 in the Appendix compares the series for profits as a fraction of GDP predicted by the model with that observed for the U.S. economy. The prediction of the model matches relatively well both the volatility and cyclicality of the profit series, even though the data for profits was not used in the estimation of the model.

Table 3. Posterior Estimates

		Prior	Posterior Mode			
Parameter		Mean	0. BM	1. K=0	2. H-M	3. Shimer
Matching elasticity	α	0.5	0.65	0.12	0.5*	0.72*
Bargaining power of worker	Ψ	0.5	0.52	0.26	0.08	0.72*
Unemployment benefit	b	0.4	0.41	0.44	0.956	0.40*
Job destruction rate	s	0.04	.038	.038*	.038*	.038*
Fraction of training costs	φ	0.5	0.94	0.0*	0.0*	0.0*
Total costs	μ	0.3	0.32	0.21	0.044	0.6*
Persistence of productivity	ρ_A	0.5	0.89	0.89	0.88	0.82
Marginal Data Density	MD		3815	3551	3515	3212

Asterisks (*) indicate that a parameter was fixed at the corresponding value.

employment pool two periods later. As shown in the Appendix in Figures 13 and 14, this assumption improves the fit of job creation substantially. However, the measure of overall fit based on marginal data density shows little improvement. This indicates that fitting observed modest variations in job creation rates is not important for understanding the behavior of other time series. This is not at all surprising given the inconsistencies between different measures of job creation discussed in Section II, and given the fact that job creation is uncorrelated with most other variables in the data.

The gap between wages in the model and in the data is satisfactory given that the discrepancy between the two commonly used series for real wages is large. The two commonly used series for real wages are average hourly earnings in private nonfarm payrolls divided by the consumption price index and the labor share times labor productivity. The root mean square difference between the two detrended series is 0.94 log points which is comparable to average wage variability over the cycle of 0.97 log points. The series for labor productivity also have a large measurement error component.

Table 4. Explanatory Power of Alternative Models

	Fraction of Variations Explained								Δ MD
	Y	U	V	JD	JC	JF	W	Y/N	
0. Benchmark	.62	.75	.99	.81	.50	.98	.41	.41	603
1. K=0	.43	.99	.08	.43	.11	.60	.42	.41	339
2. H-M	.54	.54	.71	.00	.93	.96	.30	.47	303
3. Shimer	.47	.07	.08	.00	.11	.06	.70	.69	0

To study the importance of the two key assumptions for model performance, I compare the performance of the benchmark model with three alternative specifications. In the first specification, I set training costs to zero and re-estimate the model. In the second specification, I apply the same estimation strategy to Shimer's model allowing for variations in the value of unemployment benefits and the bargaining weight.[23] I denote this specification "H-M" because the resulting estimates replicate the calibration of Hagedorn and Manovskii (2008). The last specification is Shimer's original calibration.

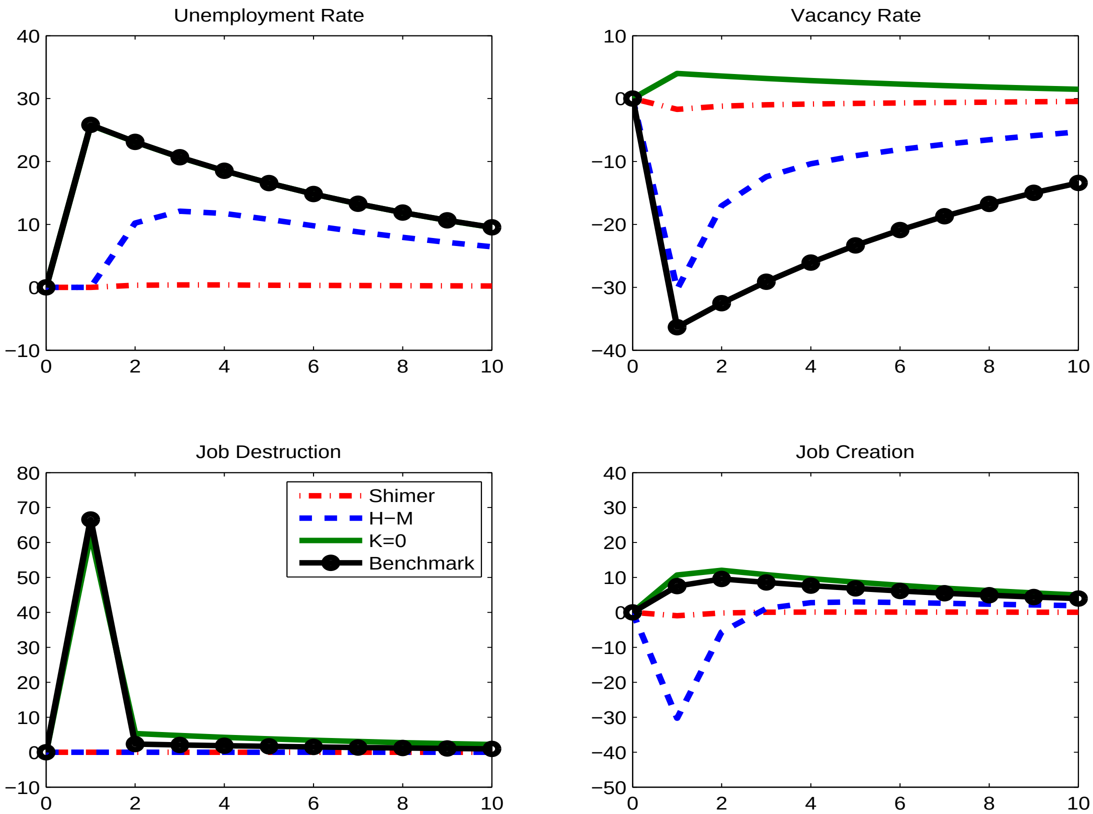
Table 3 presents the posterior modes of parameters for all four specifications. Asterisks indicate parameters for which values were fixed. The benchmark model (BM) has all the parameters estimated. The first alternative specification fixes the fraction of training costs to zero and the steady-state job destruction rate to its previously estimated value. The second alternative specification (H-M) exploits Shimer's model with the matching elasticity set to 0.5.

The estimated values of unemployment benefits of 0.956 and bargaining power of 0.08 are very similar to those obtained through a calibration exercise by Hagedorn and Manovskii (2008). The last specification fixes most parameters to values adopted by Shimer (2005). The last row of Table 3 reports the value of marginal data density for each model.[24] These values represent the log-likelihood of the same set of data

[23]The model used for this exercise is described in the Appendix and follows closely the model in Shimer (2005).

[24]Marginal data density is a Bayesian analog of the Bayes information criterion, a robust means of model comparison, which uses the likelihood function as a measure of fit and penalizes the model

FIGURE 7. Impulse Responses to a 1 St.Dev. Negative Shock.



given each model of interest. They show that each of the two key elements of the benchmark model helps improve the its ability to fit the data.

Table 4 compares the fit of the four models in more detail. Numbers in the rows of Table 4 describe fractions of standard deviations of the data explained by the three alternative specifications and the benchmark specification. The last column computes the gain in marginal data density of each model compared to Shimer's original calibration.

Comparison of lines 2 and 3 indicates that the calibration of Hagedorn and Manovskii, indeed, improves the performance of the labor search model, explaining half of variations in unemployment and 70 percent of variations in vacancies. However, it still

for over-parametrization. A gain in marginal data density is equivalent to the log of the posterior odds ratio.

falls substantially short of providing a good fit of the data. A model with match-specific heterogeneity but without the job destruction margin or training costs would have implications similar to that of line 2.

Comparison of line 1 to line 3 and of line 0 to line 2 demonstrates that explaining variations in job destruction enhances the ability of the model to capture the behavior of unemployment, accounting for its initial increase during recessions. Comparison of line 1 to line 0 shows that incorporating training costs is crucial for explaining the decrease in vacancies and the modest response of job creation. Thus, both the job destruction margin and training costs are key to the empirical performance of the benchmark model.

The explanatory power added by these two elements is two times larger than that produced by the H-M calibration. However the benchmark specification does not rely on the two most controversial assumptions: a low value of total job creation costs and a low value of bargaining power of the worker. Instead, it matches very well empirical values for both the total job creation costs and the split of these costs into the recruiting and training components. It also uses a more conventional value of unemployment benefits, while the effect of assumptions about the value of bargaining power on model fit is negligible.

To illustrate why the benchmark model outperforms its predecessors it is useful to compare impulse response functions to a recessionary shock. Figure 7 compares impulse responses of the four model specifications. Shimer's calibration generates almost no response to a recessionary shock. The calibration of Hagedorn and Manovskii explains about half of the response of unemployment, all of it through the job creation margin. The low explanatory power of this specification indicates that the job creation margin alone is not powerful enough to fit variations observed in the data. Although simulations of a calibrated version of model 2 seem to fit well the restricted set of moments of the data, my result demonstrates that the calibration exercise omits important information about labor market dynamics. A likelihood approach which takes into account all of the available information shows that the explanatory power of a calibrated version of model 2 is low.

As demonstrated by estimates of model 1, an introduction of the job destruction margin alone can help explain a large fraction of fluctuations in unemployment, but is less satisfactory at explaining the behavior of vacancies. This is because large increases in job destruction in absence of training costs motivate firms to create more vacancies in a recession. In fact, model 1 generates countercyclical vacancy rates, while they are procyclical in the data. The benchmark model explains, both quantitatively and qualitatively, a large fraction of the observed fluctuations in all of the variables of interest.

VI. Extensions

In this part, I consider two major extensions of the benchmark model. First, I extend the model to allow for persistent taste shocks. Second, in the framework with persistent taste shocks, I consider job-to-job transitions by allowing workers with low-paying jobs to search for better paying jobs. This specification generates highly plausible out-of-sample predictions for the behavior of quits. In addition, I re-evaluate the performance of the benchmark model using data on worker flows instead of job flows. This exercise reinforces the main findings of the paper by showing that they are robust to the choice of data on labor market transition rates.

Mild extensions of the model, such as allowing for a more flexible structure of household preferences, as well as incorporating an exogenous job destruction component, do not improve the performance of the model substantially. Since parameters of these specifications cannot be separately identified using the available data, I relegate their description to the Appendix and do not consider them here. The same statement can be made regarding firing costs, which have an effect indistinguishable from an increase in fixed costs of job creation.

VI.1. **Persistent taste shocks.** To model persistent taste shocks, I replace the specification (6) by a more general specification. This specification assumes that new matches are born with $i = 0$, i.e. start producing at the production possibility frontier. With probability p, existing matches are shifted away from the frontier by new entrants. Alternatively, with probability $1 - p$, they draw new realizations of the taste shock from the same uniform distribution. The stationary distribution of taste shocks under the new specification remains uniform, as in the benchmark specification. The parameter p represents persistence of taste shocks, thus, allowing for persistent individual histories. This specification preserves the nice properties of the benchmark specification, where the number of jobs is the state variable which fully characterizes the productivity distribution. If the persistence parameter is set to zero, the specification boils down to the benchmark specification. The process for taste shocks is assumed to be drawn from:

$$z_{it} = e^{-gi}, \qquad i_{t+1}|i_t = \begin{cases} i_t + M_{t+1}, & p \\ \mathcal{U}\left[0, J_{t+1}\right], & 1-p \end{cases} \tag{26}$$

When individual histories are persistent, the value of the match (17) also becomes history-dependent. In this case, the value function of a match has a closed-form solution with respect to its index i, which can be obtained using the method of undetermined coefficients. The dynamic equation describing the evolution of this value function can be approximated, to the first order, by the following equation:

$$\Gamma_{i,t} = E_t \frac{\beta}{1 - p\beta e^{-gM_{t+1}}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \left[\begin{array}{c} pe^{-gM_{t+1}} A_{t+1} \left(e^{-gi} - e^{-gN_t}\right) \\ +(1-p)\left(\frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \Gamma_{N,t+1}\right) \end{array} \right], \tag{27}$$

where the value of a match at the job destruction cutoff, $\Gamma_{N,t}$, is defined similarly to equation (18) in the benchmark model:

$$A_t e^{-gN_t} - w_t + \Gamma_{N,t} = 0. \tag{28}$$

The main new implication of equation (27) comes from the fact that it draws a difference between the value of a match at destruction, $\Gamma_{N,t}$, and the value of a new match, $\Gamma_{0,t}$, which affects job creation through equations (13) and (19). Depending on persistence of taste shocks, p, and the curvature of the distribution, g, implied by job creation costs, μ, the model with persistent shocks allows for much richer dynamic behavior of job creation and job destruction rates in response to aggregate shocks. The rest of the equations of the model remain the same.

TABLE 5. Posterior Estimates

		Prior	Posterior Mode		
Parameter		Mean	BM	Ext. 1	Ext. 2.
Matching elasticity	α	0.5	0.65	0.65	0.27
Bargaining power of worker	Ψ	0.5	0.52	0.36	0.41
Unemployment benefit	b	0.4	0.41	0.45	0.43
Job destruction rate	s	0.04	.038	.038	.038*
Fraction of training costs	φ	0.5	0.94	0.94	0.81
Total costs	μ	0.3	0.32	0.32*	0.32*
Persistence of productivity	ρ_A	0.5	0.89	0.89	0.89
Persistence of taste	p	0.5	0.0*	0.33	0.42
Fraction of training costs	φ_Q	0.5	—	—	0.93
Marginal Data Density	MD		3815	3814	3817

Asterisks (*) indicate that a parameter was fixed at the corresponding value.

When estimating the extended model on the same set of data, it appears that the difference in fit is negligible. The posterior on the persistence of taste shocks has the mean of 0.33. The posterior is not much narrower than the prior, which indicates that, given the available data, taste persistence is not identified separately from the total cost parameter and the bargaining power parameter. The posterior estimates of other parameters for this specification are presented in column 'Ext. 1' in Table 5.

The fact that the parameter p does not have a significant effect on the results indicates that the main findings of the paper based on the benchmark specification are robust to alternative assumptions about the behavior of taste shocks. This intuition is confirmed by the fact that posterior distributions of all other estimated parameters are very similar to those obtained in the estimation of the benchmark model.

VI.2. **Job-to-job transitions.** The second extension I consider makes use of the specification with persistent taste shocks. In this extension, I allow workers already matched with jobs to compete with unemployed workers in search of better-paying jobs. Because search is costly, this option will only be attractive to employed workers who are close to the bottom of the productivity distribution. If matched with a vacant position, such workers would quit their existing employment relationships and move to the top of the productivity distribution.

In this specification, depending on costs of search on-the-job, there will be a second cutoff, $N_{Q,t}$, and a corresponding value of a match, $\Gamma_{Q,t}$, such that only workers with $i \in [N_{Q,t}, N_t]$ will find it profitable to search for more productive jobs. The matching function (3) will match vacancies V_t not just with unemployed workers, U_t, but with the sum of unemployed workers and potential quitters, $1 - N_{Q,t}$.

A worker at the second cutoff must be exactly indifferent whether to search for a better paying job and incur the cost of search, or not to search. The value at the cutoff must then satisfy:

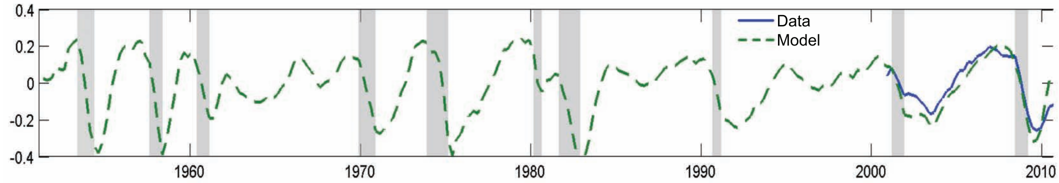
$$(\Gamma_{0,t} - \Gamma_{Q,t} - K_Q) \frac{Q_t}{N_t - N_{Q,t}} = f \tag{29}$$

where Q_t is the number of quits, f is the cost of applying for a job, and K_Q is the fixed cost of training to do the new job. Here, for simplicity, it is assumed that all the cost of job creation is incurred by the worker. This is not a very restrictive assumption, since the fixed cost of job creation is allowed to differ from that incurred by unemployed workers.

$$\frac{Q_t}{N_t - N_{Q,t}} = \frac{M_t}{1 - N_{Q,t}} \tag{30}$$

I assume that matching is random, implying a proportional split of new matches between unemployed workers and potential job quitters, as represented by equation (30). Because the total number of new matches per period is higher in the model with job-to-job transitions, existing jobs are shifted away from the frontier faster in this model. As a result, job-to-job transitions lead to a net exchange of jobs among workers that were not laid off due to low productivity. When workers simply exchange jobs, this does not lead to observable changes in job creation or job destruction rates. Thus, the aggregate job destruction and job creation rates in this model are affected

FIGURE 8. Fit of Quits.



only indirectly by the change in incentives that the possibility of a quit has on the behavior of workers and on matching patterns.

To calibrate this model I use the fact that the cost of applying to a job in steady-state maps one-to-one into the steady-state quit rate. I calibrate the steady-state quit rate to 7 percent, its average quarterly value in the U.S. The only new parameter to be estimated is then the proportion of fixed costs in the total cost of changing a job, which I denote $\varphi_Q = \frac{Q_{ss}K_Q}{\left(N_{ss}-N_{Q,ss}\right)f+Q_{ss}K_Q}$.

Table 5 describes the estimated parameters for this new specification in column 'Ext. 2'. The main difference from the benchmark specification is in the estimate of the matching elasticity, which is now in the neighborhood of 0.3. The estimates of shares of training costs are both relatively high, consistent with micro estimates of these costs. The incorporation of job-to-job transitions into the model improved its overall fit only marginally, while accounting well for the volatility of quits in JOLTS data, as shown in Figure 8[25].

VI.3. **Alternative data on worker flows.** In addition to two extensions, I consider an alternative source of data on worker flows. Instead of using data on job flows from the establishment survey, I use data on worker flows constructed from household survey data. More specifically, I replace data on job destruction, job creation and the job finding rate used in the main estimation exercise by data constructed from the separation rate and the job finding rate computed by Fujita and Ramey (2006).

The main difference in the statistical properties of data on worker flows (shown in Table 6) is that the separation rate is slightly less volatile than the job destruction rate, and the correlation structure of job creation rates with the business cycle is slightly different. A disadvantage of this data is that it is available for a shorter time

[25]Data on quits was not used to estimate the parameters of the model.

Table 6. Comparison of Second Moments

Standard Deviations								
	Y	U	V	JD	JC	JF	W	Y/N
Data	1.33	11.6	13.0	10.4	6.6	7.3	0.96	0.95
Model	0.95	10.2	12.9	7.6	6.7	7.3	0.37	0.56
Correlations with GDP								
Data	1	-0.83	0.85	-0.75	-0.06	0.76	-0.02	0.47
Model	1	-0.99	0.99	-0.87	-0.99	0.99	0.99	0.99

Table 7. Posterior Estimates

		Prior	Posterior Mode			
Parameter		Mean	0. BM	1. K=0	2. H-M	3. Shimer
Matching elasticity	α	0.5	0.68	0.10	0.5*	0.72*
Bargaining power of worker	Ψ	0.5	0.34	0.06	0.05	0.72*
Unemployment benefit	b	0.4	0.46	0.50	0.957	0.40*
Job destruction rate	s	0.04	.06	.06*	.06*	.06*
Fraction of training costs	φ	0.5	0.94	0.0*	0.0*	0.0*
Total costs	μ	0.3	0.32	0.31	0.043	0.6*
Persistence of productivity	ρ_A	0.5	0.91	0.91	0.91	0.87
Marginal Data Density	MD		2004	1835	1807	1692

Asterisks (*) indicate that a parameter was fixed at the corresponding value.

Table 8. Explanatory Power of Alternative Models

	Fraction of Variations Explained								Δ MD
	Y	U	V	JD	JC	JF	W	Y/N	
0. Benchmark	.72	.88	.99	.72	.86	.99	.36	.37	312
1. K=0	.76	.94	.21	.76	.87	.80	.33	.35	143
2. H-M	.66	.62	.66	.00	.55	.96	.28	.48	115
3. Shimer	.58	.13	.15	.00	.04	.12	.74	.77	0

period. Because worker transition rates are only available for the period from 1976:I to 2005:IV, I restrict all other data series used in the estimation procedure to a subset of the same length.

To do a full robustness check of the main results, I re-estimate all four models of interest using this alternative data source. The results of the estimation exercise are presented in Tables 6, 7 and 8. A comparison with Tables 2, 3 and 4 respectively indicates that the shorter data set using data on worker flows does not just confirm the main results of the paper, but rather reinforces them. The benchmark model explains a bigger fraction of variations in the variables of interest, while the parameter estimates are essentially the same. The only parameter estimate that is slightly different is the steady-state job destruction rate. A higher value of the job destruction rate is a direct consequence of the slightly lower volatility of separations in the data.

The model is able to fit better both the unemployment rate and the job creation rate derived from worker flow data, while the fit of other variables remains virtually the same. A slightly lower fraction of unemployment fluctuations is attributed to job destruction, which is also due to a lower amount of fluctuations in the separation rate. The results of this robustness check indicate that the mechanisms of the model capture well the dynamic properties of the labor market independent of the source of data on worker and/or job flows used to study it.

VII. Conclusion

In this paper, I emphasize two elements of the Mortensen-Pissarides model: the job destruction margin and training costs. I show that these two elements and the interaction between them enable the model to explain the sharp increases in unemployment and the large declines in job availability in recessions. I embed these two key elements into a general equilibrium model with a matching friction. Such a model, driven by a single aggregate shock, can simultaneously explain most variations in unemployment, vacancies, job creation, job destruction and job finding rates, while remaining consistent with variability and cyclicality of profits, labor productivity and real wages. I estimate parameter values that provide the best fit of the data and find that they are all of plausible magnitude. I estimate an extension of the model, allowing for persistent idiosyncratic shocks and, hence, providing incentives for on-the-job search. This extended version of the model is found to be consistent with observed fluctuations in job-to-job transitions in the U.S. economy. Using data on worker flows instead of job flows only reinforces the results.

Obviously, there are many features of the real world that these models do not address. Desirable extensions include understanding the role of capital formation and the vintage effects of matches between capital and labor. Another important direction of further research is a more plausible specification of the job creation process that allows for additional delays and takes into account detailed microeconomic studies of creation costs.[26]

Two major puzzles relevant for analyzing labor market policies remain to be resolved. First, it would be helpful to understand the determinants of the labor wedge, which in a compact way represents the sum of labor market distortions. Second, when calibrated to match unemployment volatility, the majority of existing models of the labor market make the unemployment rate too sensitive to variations in the size of unemployment benefits. The version of the MP model considered in this paper goes a long way toward explaining labor market fluctuations in the U.S. in the last fifty years, and, thus, can serve as a useful starting point not only for quantitative explorations of the effects of labor market policies, but also for shedding light on these open questions.

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[26]Cross-country analysis of job creation costs and their consequences for the slope of the Beveridge curve is another potential research topic.

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VIII. Appendix (not for publication)

VIII.1. **Computation of the Steady-state.** For the general derivation I use a general specification of preferences:

$$u\left(C_t, N_t\right) = \frac{C_t^{1-\gamma}-1}{1-\gamma} - \varpi\frac{\left(N_t\right)^{1+\eta}}{1+\eta}.$$

I augment the model to allow for constant exogenous job destruction d in addition to endogenous job destruction ζ_t.

The system of equations of the model can then be reduced to:

(1) $A_t = A_{ss}^{1-\rho_A} A_{t-1}^{\rho_A} e^{\sigma_A \varepsilon_t} \qquad \varepsilon_t \in \mathcal{N}\left(0,1\right)$

(2) $M_t = BU_t^{\alpha} V_t^{1-\alpha}$

(3) $U_t = 1 - N_t$

(4) $J_{t+1} = N_t + M_t$

(5) $N_t = J_t\left(1-\zeta_t\right)\left(1-d\right)$

(6) $Y_t = A_t \frac{1-e^{-gN_t}}{g}$

(7) $C_t = Y_t + bU_t - X_t$

(8) $w_t = \frac{\varpi (N_t)^{\eta}}{C_t^{-\gamma}} + b + \Psi\left(\Gamma_t - K\right)\frac{M_t}{U_t}$

(9) $c = \left(1-\Psi\right)\left(\Gamma_t - K\right)\frac{M_t}{V_t}$

(10) $\Gamma_t = \left(1-d\right) E_t \beta \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\left(\frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \Gamma_{t+1}\right)$

(11) $A_t e^{-g\frac{N_t}{1-d}} - w_t + \Gamma_t = 0$

(12) $X_t = cV_t + KM_t$

(13) $W_t = \Psi\frac{Y_t}{N_t} + \left(1-\Psi\right) w_t$

(14) $\frac{\Pi_t}{Y_t} = \left(1-\Psi\right)\left(1 - \frac{w_t N_t}{Y_t}\right)$

where $\{A_t, J_t, N_t, M_t, U_t, X_t, V_t, Y_t, C_t, \Gamma_t, \zeta_t, w_t, W_t, \Pi_t\}$ are the endogenous variables of the model and ε_t is the exogenous shock.

The steady-state of the model is computed as follows. First, elasticity parameters of the model are set: $\{\beta, \gamma, \eta, \Psi, \alpha, d\}$. Then, the scales of output and labor are defined by setting $A_{ss} = 1$ and $U_{ss} = u$.

Steady-state values $\left\{\mu = \frac{X_{ss}}{W_{ss}M_{ss}}, \frac{b}{A_{ss}}, \varphi = \frac{KM_{ss}(1-\Psi)}{cV_{ss}+KM_{ss}(1-\Psi)}, vf = \frac{M_{ss}}{V_{ss}}, s = \zeta_{ss}\right\}$ map uniquely into parameters $\{g, \varpi, c, K, B\}$. Parameters $\{\rho_A, \sigma_A\}$ characterize the properties of exogenous shocks.

The algorithm to compute the parameters of the model and its steady-state values conditional on the set of parameters $\{\beta, \gamma, \eta, \Psi, \alpha, d, \mu, b, \varphi, u, s, \rho_A, \sigma_A, A, vf\}$ uses the following steps:

a) $N = 1 - u, \qquad M = \frac{s+d-sd}{1-d}\frac{N}{1-s}, \qquad J = M + N, \qquad \varphi_2 = \frac{\varphi}{(1-\varphi)(1-\Psi)+\varphi}.$

b) Use a numerical solver to find g that satisfies $\frac{e^{gN}-1}{gN} = \frac{\beta(1-d)\left(\frac{1}{\mu(1-\Psi\varphi_2)}-1\right)+1}{\beta(1-d)\left(\frac{1}{\mu(1-\Psi\varphi_2)}-1\right)-\frac{\Psi}{1-\Psi}}.$

c) $\frac{w}{A} = e^{-g\frac{N}{1-d}} + \beta(1-d)\frac{1-e^{-gN}+gNe^{-g\frac{N}{1-d}}}{gN}, \qquad \frac{\Gamma}{A} = \beta(1-d)\frac{1-e^{-gN}+gNe^{-g\frac{N}{1-d}}}{gN},$

d) $\frac{Y}{A} = \frac{1-e^{-gN}}{g}, \qquad \frac{C}{A} = \frac{Y}{A} - \frac{1-\Psi}{1-\Psi\varphi_2}M\frac{\Gamma}{A} + \frac{b}{A}U$

e) $\varpi = \frac{\frac{w}{A} - \Psi\frac{1-\varphi_2}{1-\Psi\varphi_2}\frac{M}{U}\frac{\Gamma}{A} - \frac{b}{A}}{N^\eta\left(\frac{C}{A}\right)^\gamma A^{\gamma-1}}, \qquad X = \frac{1-\Psi}{1-\Psi\varphi_2}\frac{\Gamma}{A}MA,$

f) $K = \varphi_2\frac{X}{M}, \qquad V = vf * M,$

g) $c = (1-\varphi_2)\frac{1-\Psi}{1-\Psi\varphi_2}\frac{\Gamma}{A}\frac{M}{V}A, \qquad B = \frac{M}{U^\alpha V^{1-\alpha}}$

VIII.2. **Derivation of the Beveridge Curve.** I linearize the reduced system of equations above to obtain:

(1′) $a_t = \rho_A a_{t-1} + \sigma_A \varepsilon_t$

(2′) $m_t = \alpha u_t + (1-\alpha) v_t$

(3′) $n_t = -\kappa u_t$

(4′) $j_{t+1} = (1-s) n_t + s m_t$

(5′) $n_t = j_t - \frac{s}{1-s} s_t$

(6′) $y_t = a_t + p n_t$

(7′) $d_c c_t + d_x x_t = d_y y_t + d_u u_t$

(8′) $\eta n_t - \gamma c_t = f_w w_t - f_v (v_t - u_t)$

(9′) $g_t = (1-\varphi)(v_t - m_t)$

(10′) $g_t = -\gamma c_{t+1} + \gamma c_t + k_y (y_{t+1} - n_{t+1}) - k_a (a_t - \delta n_t)$

(11′) $a_t - \delta n_t = h_w w_t - h_g g_t$

(12′) $x_t = (1-\varphi_2) v_t + \varphi_2 m_t$

where I introduce new notation:

$\kappa = \frac{u}{1-u} \qquad p = \frac{gNe^{-gN}}{1-e^{-gN}} \qquad \delta = gN \qquad h_g = \frac{\Gamma}{w-\Gamma} \qquad h_w = \frac{w}{w-\Gamma}$

$d_y = \frac{Y}{Y-X+bU}$ $\quad d_x = \frac{X}{Y-X+bU}$ $\quad d_u = \frac{bU}{Y-X+bU}$

$f_w = \frac{w}{w-b-\frac{\Psi}{1-\Psi}c\frac{V}{U}}$ $\quad f_v = \frac{\frac{\Psi}{1-\Psi}c\frac{V}{U}}{w-b-\frac{\Psi}{1-\Psi}c\frac{V}{U}}$ $\quad k_y = \frac{\frac{Y}{N}}{\frac{Y}{N}-Ae^{-gN}}$ $\quad k_a = \frac{Ae^{-gN}}{\frac{Y}{N}-Ae^{-gN}}$

I then sequentially substitute out linearized equations to derive the Beveridge curve:

$(a) \qquad j_{t+1} = (1-s)\, n_t + s m_t$

$(b) \qquad n_t = j_t - \frac{s}{1-s} s_t$

$(c) \qquad m_t = \alpha u_t + (1-\alpha)\, v_t$

$(d) \qquad x_t = u_t + (1-\varphi_2\alpha)\,(v_t - u_t)$

$(e) \qquad n_t = -\kappa u_t$

$(f) \qquad y_t = a_t - p\kappa u_t$

$(g) \qquad h_w w_t = a_t + \delta\kappa u_t + h_g g_t$

$(h) \qquad (v_t - u_t) = \frac{1}{(1-\varphi)\alpha} g_t$

$(i) \qquad a_t = \rho_A a_{t-1} + \sigma_A \varepsilon_t$

$(j) \qquad c_t = d_y a_t - d_x \frac{1-\varphi_2\alpha}{(1-\varphi)\alpha} g_t + (d_u - d_x - d_y p\kappa)\, u_t$

$(k) \qquad g_t = \gamma\,(1-\rho_A)\, c_t + \rho_A k_y\,(a_t + (1-p)\,\kappa u_t) - k_a\,(a_t + \delta\kappa u_t)$

This leaves us with two equations:

$$G_1 g_t = A_1 a_t + U_1 u_t$$
$$G_2 g_t = A_2 a_t + U_2 u_t$$

where $\quad A_1 = (f_w + h_w\gamma d_y) \qquad A_2 = (\gamma\,(1-\rho_A)\, d_y + \rho_A k_y - k_a)$

$G_1 = \left(h_w\gamma d_x \frac{1-\varphi_2\alpha}{(1-\varphi)\alpha} + h_w f_v \frac{1}{(1-\varphi)\alpha} - f_w h_g\right) \qquad G_2 = \left(1 + \gamma\,(1-\rho_A)\, d_x \frac{1-\varphi_2\alpha}{(1-\varphi)\alpha}\right)$

$U_1 = (f_w\delta\kappa + h_w\gamma\,(d_u - d_x - d_y p\kappa) + h_w\eta\kappa)$

$U_2 = (\gamma\,(1-\rho_A)\,(d_u - d_x - d_y p\kappa) + \rho_A k_y\,(1-p)\,\kappa - k_a\delta\kappa)$

These are straightforward to solve to obtain:

$\frac{u_t}{a_t} = \frac{G_2 A_1 - G_1 A_2}{U_2 G_1 - G_2 U_1} = -\frac{\Lambda}{\kappa}$

$\frac{g_t}{a_t} = \frac{A_1}{G_1} - \frac{U_1}{G_1}\frac{\Lambda}{\kappa} = \tau$

$\frac{v_t}{a_t} = \frac{1}{(1-\varphi)\alpha}\tau - \frac{\Lambda}{\kappa}$

$\frac{v_t}{u_t} = 1 - \frac{\kappa}{(1-\varphi)\alpha}\frac{\tau}{\Lambda}$

$\frac{m_t}{a_t} = \alpha\frac{u_t}{a_t} + (1-\alpha)\,\frac{v_t}{a_t}$

$\frac{s_t}{a_t} = \frac{1-s}{s}\frac{j_t - n_t}{a_t} = -\frac{1-s}{s}\Lambda$

VIII.3. **Shimer's model.** The model used by Shimer (2005) and Hagedorn and Manovskii (2008) augmented to allow for training costs can be summarized by the following two equations:

$$U_t = U_{t-1} + \zeta(1 - U_{t-1}) - BU_{t-1}^{\alpha} V_{t-1}^{1-\alpha}$$

$$(r + s + \lambda)\frac{1}{B}\left(\frac{V_t}{U_t}\right)^{\alpha} + \beta\frac{V_t}{U_t} = (1-\beta)\frac{p_t - b - (r+s)K}{c} + \lambda E_t \frac{1}{B}\left(\frac{V_{t+1}}{U_{t+1}}\right)^{\alpha}$$

Their original version is obtained by setting K to zero. The steady-state of the model is computed as follows:

$$U = u \qquad M = s(1-u) \qquad V = \frac{M}{vf} \qquad B = \frac{M}{U^{\alpha}V^{1-\alpha}}$$

$$c = (1-\beta)\frac{p - b - (r+s)K}{(r+s)\frac{V}{M} + \beta\frac{V}{U}} \qquad c = \frac{M}{V}\frac{1-\beta}{r+s+\beta\frac{M}{U}}\frac{p-b}{1+\frac{1-\beta}{1+\frac{\beta}{r+s}\frac{M}{U}}\frac{\varphi}{1-\varphi}} \qquad K = \frac{\varphi}{1-\varphi}c\frac{V}{M}$$

Parameters borrowed from the original paper are as follows: $r = 0.012$, $\lambda = 0.0635$. Parameters $p = 1$ and $vf = 0.7$ are scale parameters, which have no effect on the dynamics of the model. Estimated or calibrated parameters include bargaining power, β, unemployment benefits, b, job destruction rate, s, unemployment rate, u, matching elasticity, α, fraction of training costs, φ. These are exact counterparts of the parameters used in the benchmark model of the paper.

VIII.4. **Extensions.** The model with persistent taste shocks and job-to-job transitions is summarized by the following list of equations:

(1) $A_t = A_{ss}^{1-\rho_A} A_{t-1}^{\rho_A} e^{\sigma_A \varepsilon_t} \qquad \varepsilon_t \in \mathcal{N}(0,1)$

(2) $M_t = B(1 - N_{Qt})^{\alpha} V_t^{1-\alpha}$

(3) $U_t = 1 - N_t$

(4) $J_{t+1} = N_t + \frac{M_t}{1-N_{Qt}} U_t$

(5) $N_t = J_t(1 - \zeta_t)$

(6) $Y_t = A_t \frac{1 - e^{-gN_t}}{g}$

(7) $C_t = Y_t - cV_t - K(M_t - Q_t) - f(N_t - N_{Q_t}) - K_Q Q_t + bU_t$

(8) $w_t = \frac{\varpi N_t^{\eta}}{C_t^{-\gamma}} + b + \Psi(\Gamma_{0t} - K)\frac{M_t}{1-N_{Qt}}$

(9) $c = (1-\Psi)(\Gamma_{0,t} - K)\frac{M_t}{V_t}$

(10) $\Gamma_{N,t} = E_t \beta \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \frac{(1-p)}{1-p\beta e^{-gM_{t+1}}}\left(\frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + \Gamma_{N,t+1}\right)$

(11) $A_t e^{-gN_t} - w_t + \Gamma_{N,t} = 0$

(12) $W_t = \Psi\frac{Y_t}{N_t} + (1-\Psi) w_t$

(13) $\frac{Profit}{GDP} = (1-\Psi)\left(1 - \frac{w_t N_t}{Y_t}\right)$

TABLE 9. Moments of the Data vs Model

Moments of the Data

std	1.64	12.8	14.5	12.8	7.8	8.1	0.87	1.09
corr	Y	U	V	JD	JC	JF	W	Y/N
GDP		-0.82	0.81	-0.62	0.26	0.78	0.16	0.56
U			-0.94	0.41	0.01	-0.93	-0.11	-0.46
V				-0.50	0.07	0.92	0.16	0.49
JD					-0.61	-0.43	-0.23	-0.39
JC						0.09	0.06	0.11
JF							0.09	0.42
W								0.26

Model Generated Moments

std	1.02	9.6	14.4	10.4	3.9	8.3	0.44	0.63
corr	Y	U	V	JD	JC	JF	W	Y/N
GDP		-0.99	0.99	-0.51	-0.98	0.99	0.99	0.99
U			-0.98	0.49	0.99	-0.99	-0.99	-0.99
V				-0.51	-0.97	0.99	0.98	0.99
JD					0.39	-0.51	-0.51	-0.50
JC						-0.98	-0.97	-0.98
JF							0.99	0.99
JF								0.99

$$\Gamma_{0,t} = E_t\beta\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\frac{pe^{-gM_{t+1}}}{1-p\beta e^{-gM_{t+1}}}A_{t+1}\left(1-e^{-gN_t}\right)+\Gamma_{N,t} \tag{14}$$

$$\Gamma_{Q,t} = E_t\beta\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\frac{pe^{-gM_{t+1}}}{1-p\beta e^{-gM_{t+1}}}A_{t+1}\left(e^{-gN_{Q,t}}-e^{-gN_t}\right)+\Gamma_{N,t} \tag{15}$$

$$\left(\Gamma_{0,t}-\Gamma_{Q,t}-K\right)\frac{M_t}{1-N_{Qt}} = f \tag{16}$$

$$Q_t = \frac{M_t}{1-N_{Qt}}\left(N_t-N_{Qt}\right) \tag{17}$$

VIII.5. **Additional Tables and Figures.** The inconsistency between the data on job creation rates and the predictions of the benchmark model can be easily solved by introducing an additional assumption that matches created in period t enter the employment pool in period $t+2$. As Figures 13 and 14 demonstrate, this assumption improves the fit of job creation substantially. However, the measure of overall fit based on marginal data density shows little improvement. This indicates that fitting observed modest variations in job creation rates is not important for understanding the behavior of other time series. This is not at all surprising given that job creation is uncorrelated with most other variables in the data, as shown in Table 9. Note that in all the graphs the solid line represents the data, and the dashed line represents the fit of the model.

FIGURE 9. Comparison of Creation and Destruction for Manufacturing and All Sectors.

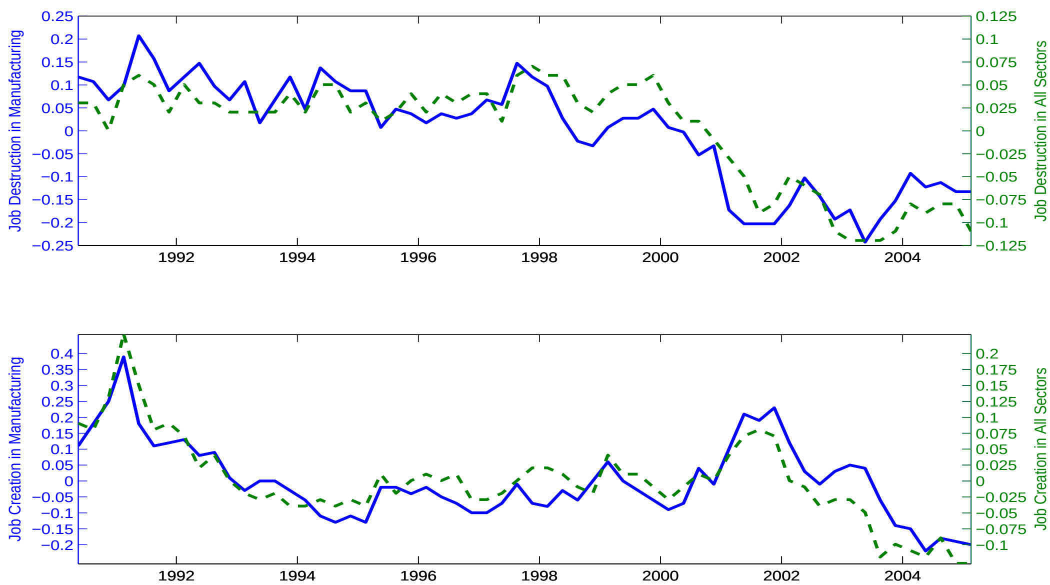

Figure 10. Fit of Output, Unemployment and Vacancies.

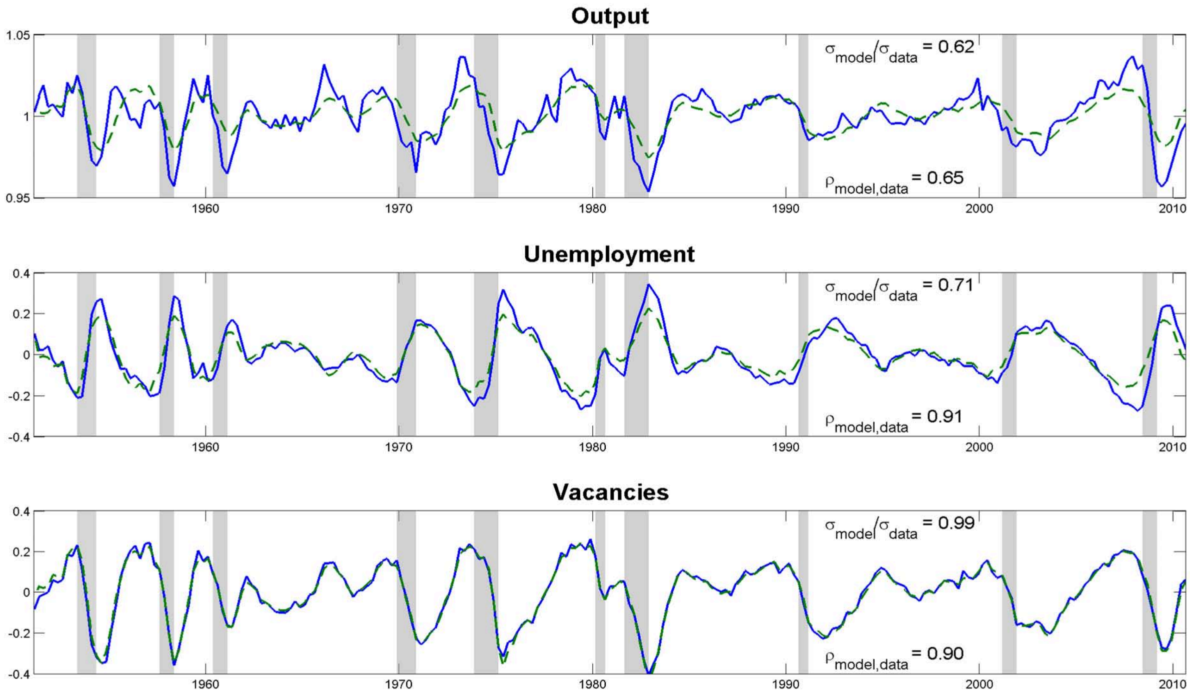

Figure 11. Fit of Job Finding, Job Destruction and Job Creation Rates.

FIGURE 12. Fit of Productivity, Wages and Profits.

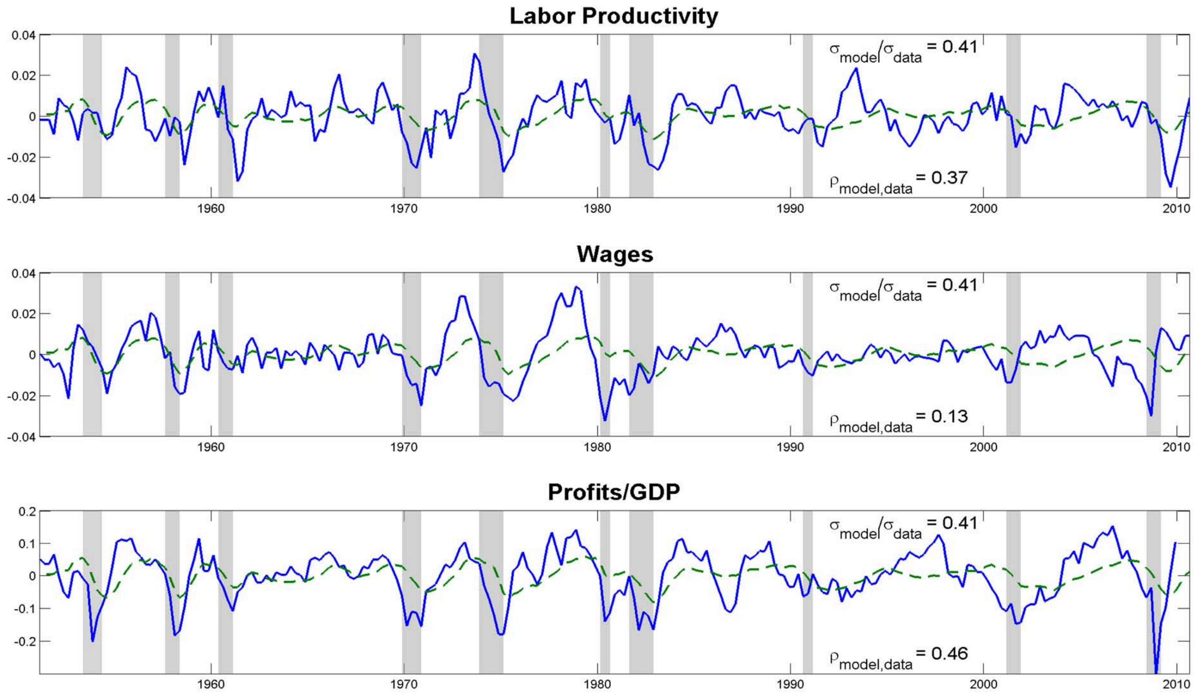

FIGURE 13. Fit of Job Creation in the Benchmark Model with Delay.

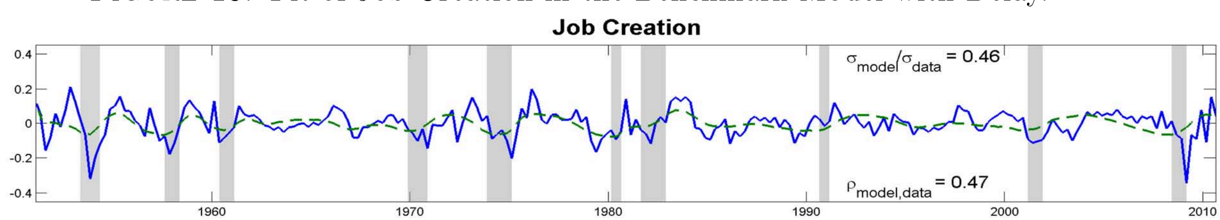

FIGURE 14. Impulse Responses with and without Delay.

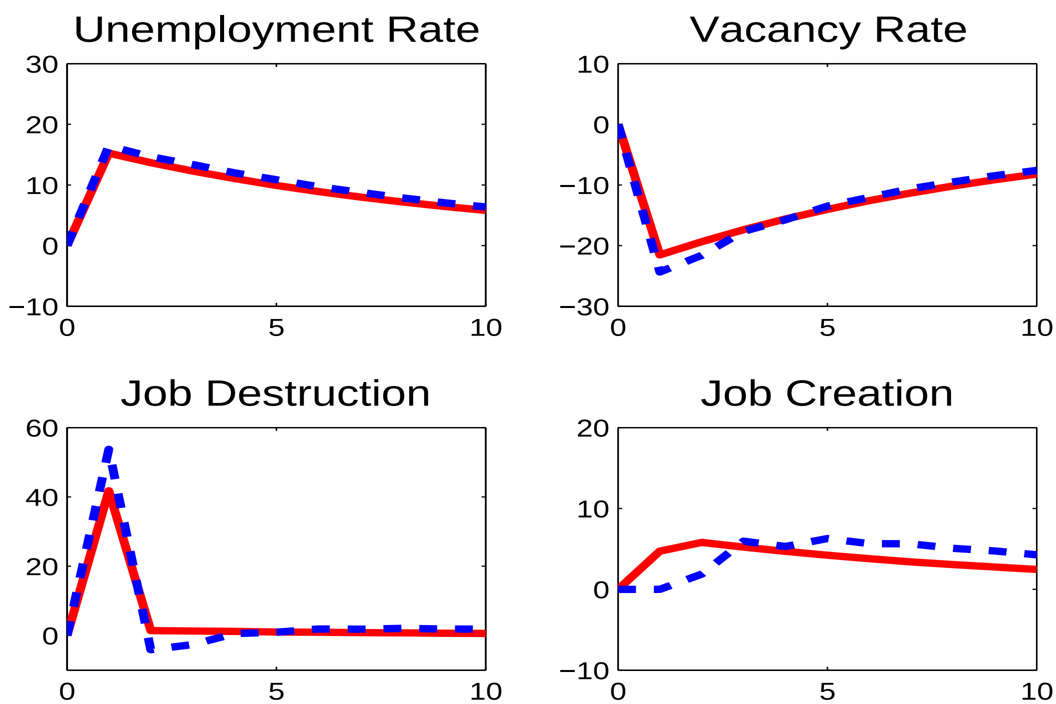

FIGURE 15. Prior and Posterior Distributions for Extension 1.

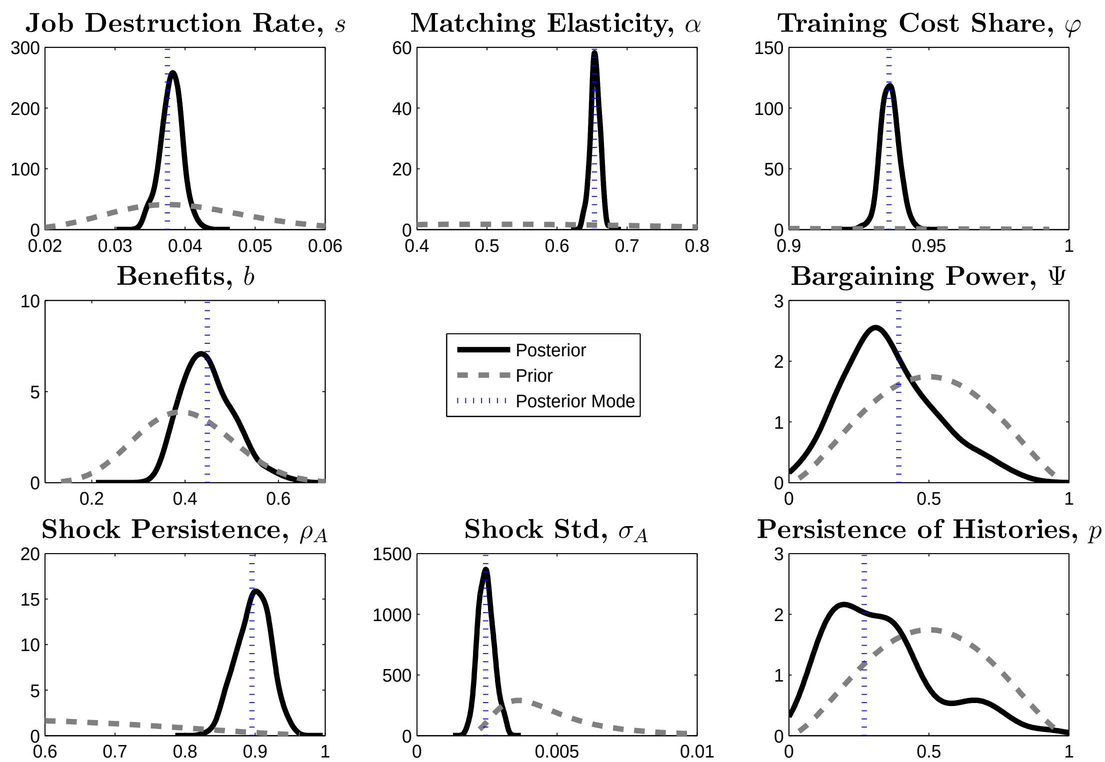

FIGURE 16. Prior and Posterior Distributions for Extension 2.

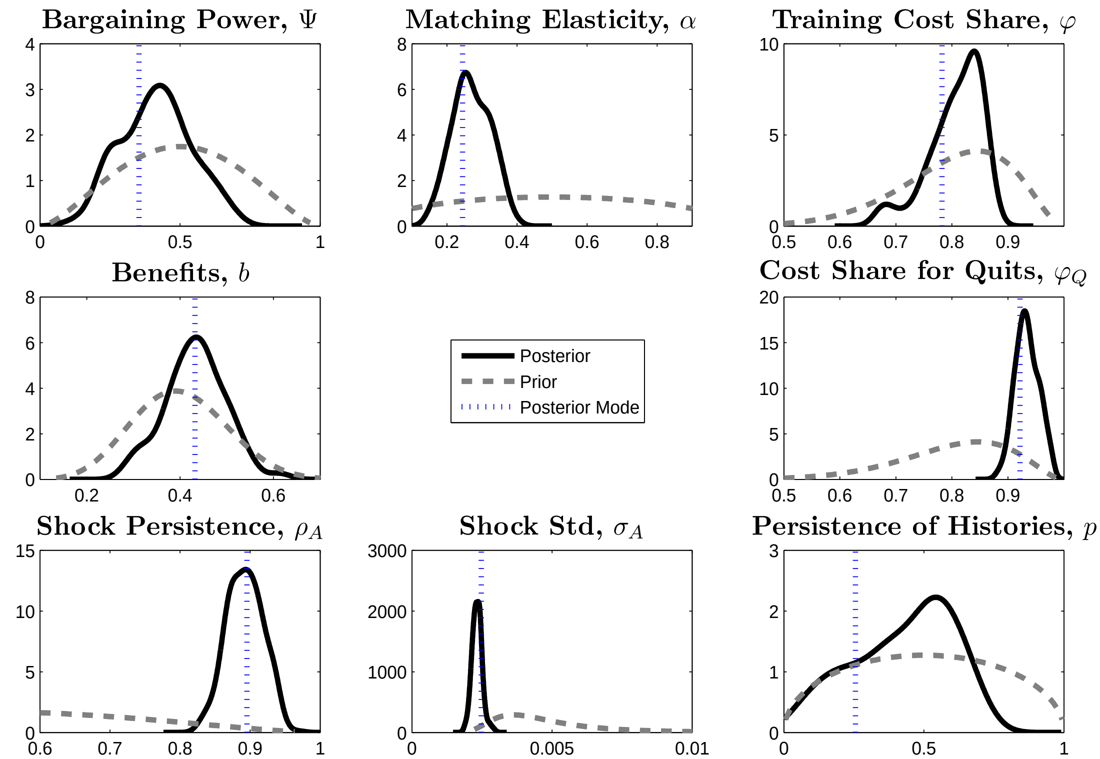

FIGURE 17. Prior and Posterior Distributions for Robustness Check.

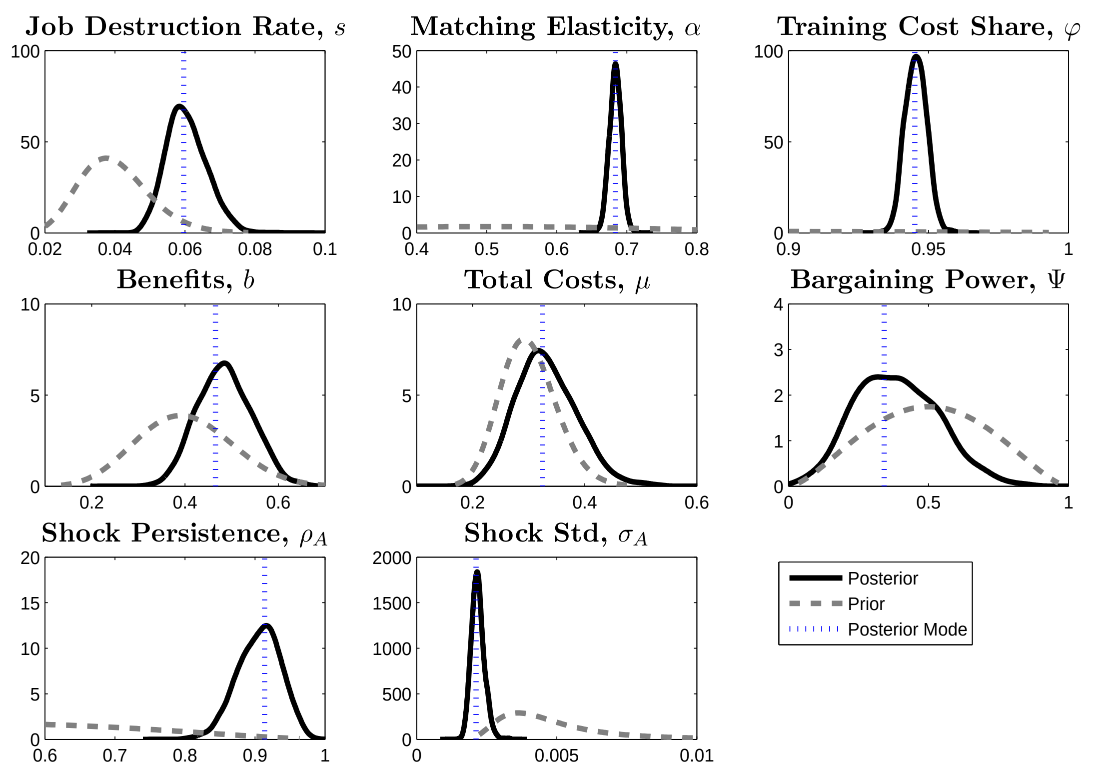

FIGURE 18. Fit of Output, Unemployment and Vacancies for the Robustness Check.

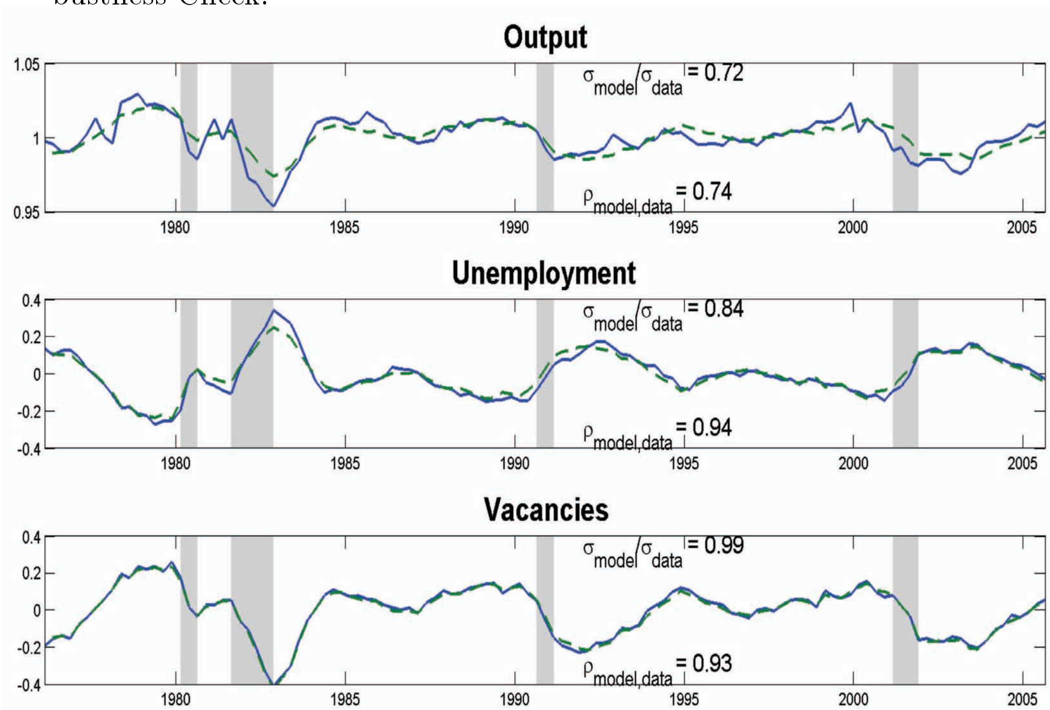

FIGURE 19. Fit of Job Finding, Job Separation and Job Creation Rates for the Robustness Check.